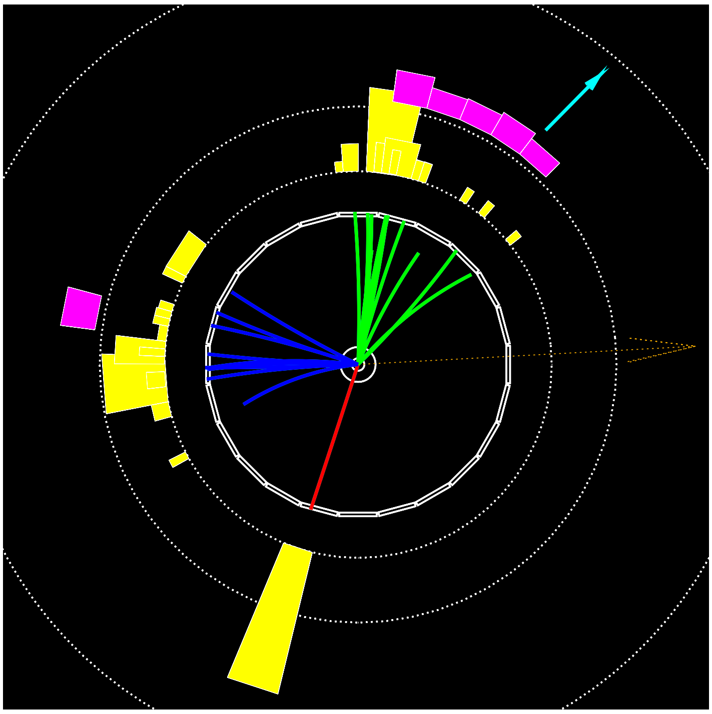
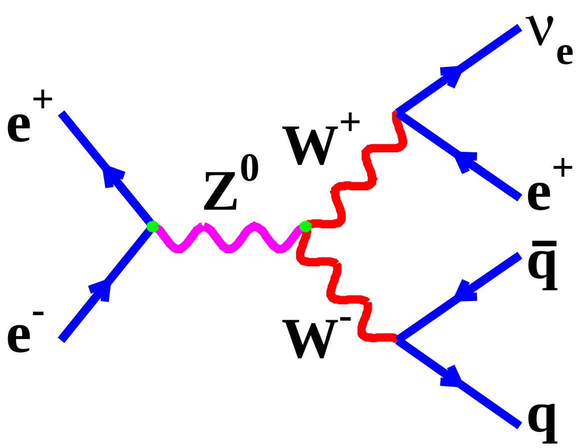
Particle Physics

Dr M.A. Thomson





Part II, Lent Term 2004

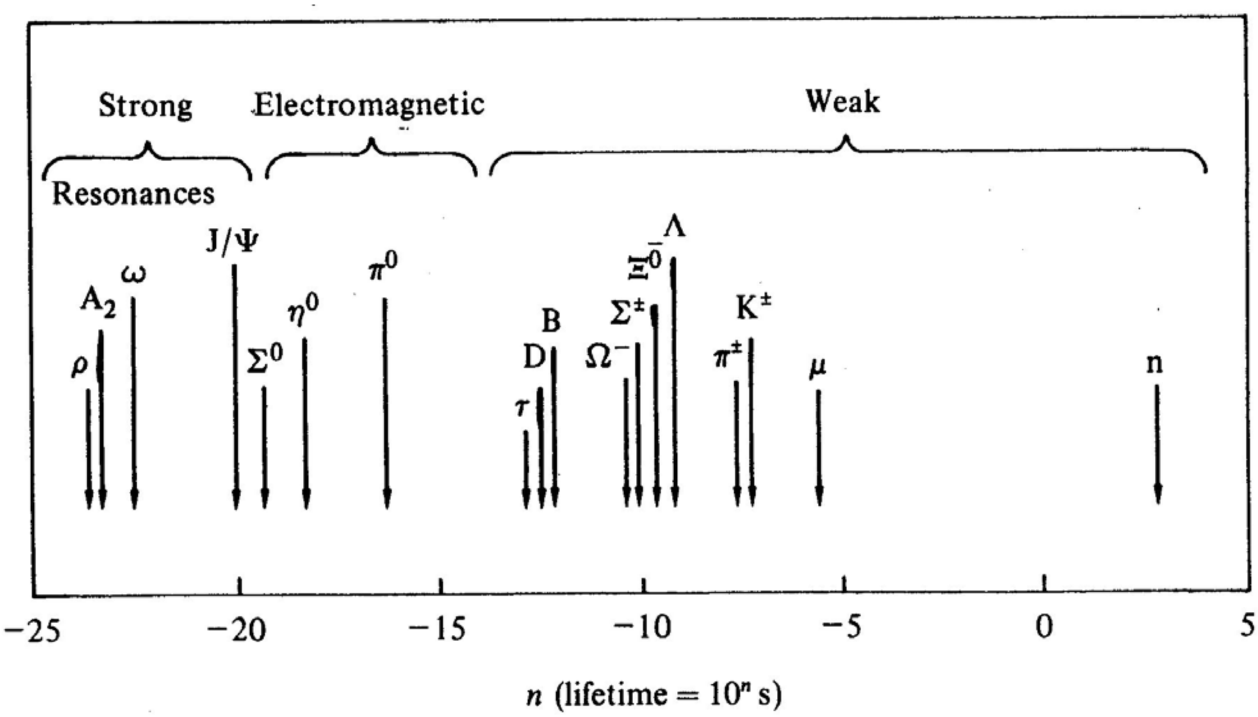
HANDOUT VI

The Weak Interaction

★ **The WEAK interaction accounts for many decays in particle physics *e.g.***

$$\mu^- \rightarrow e^-\overline{\nu}_e\nu_\mu,$$
$$\tau^- \rightarrow e^-\overline{\nu}_e\nu_\tau,$$
$$\pi^+ \rightarrow \mu^-\overline{\nu}_\mu,$$
$$n \rightarrow pe^-\overline{\nu}_e, ...$$

★ **Characterized by long lifetimes, small cross sections**



★ **Two types of WEAK interaction**

CHARGED CURRENT (CC) - $W^{\pm}$ Bosons

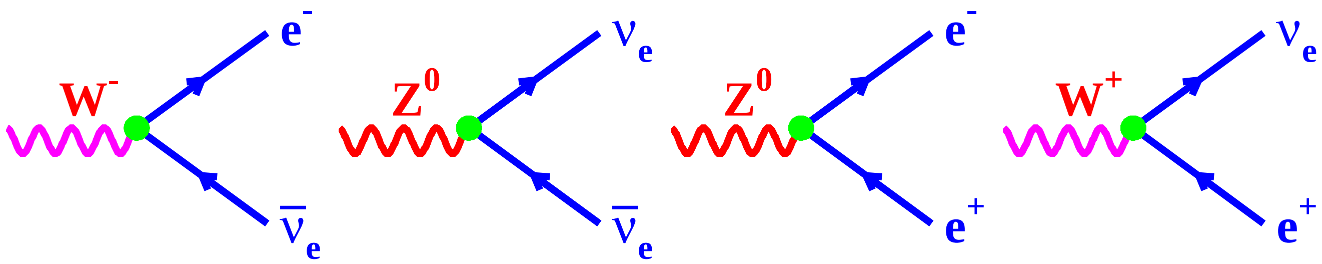
NEUTRAL CURRENT (NC) - Z^0 Boson

★ **WEAK force mediated by MASSIVE VECTOR BOSONS:**

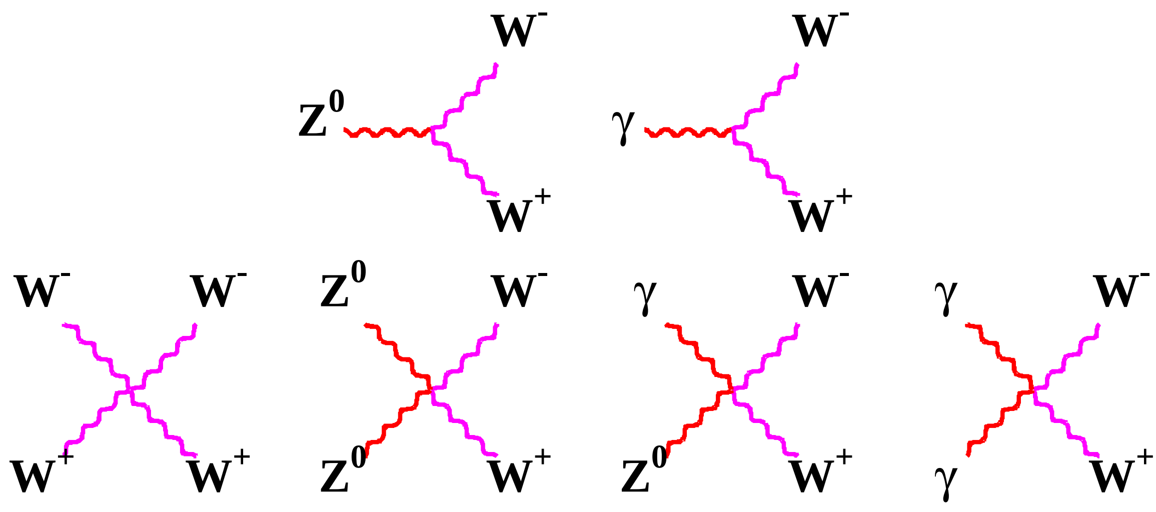
$$M_W \sim 80\ \mathrm{GeV}$$

$$M_{Z^0} \sim 90\ \mathrm{GeV}$$

★ **e.g. the WEAK interactions of electrons and electron neutrinos:**

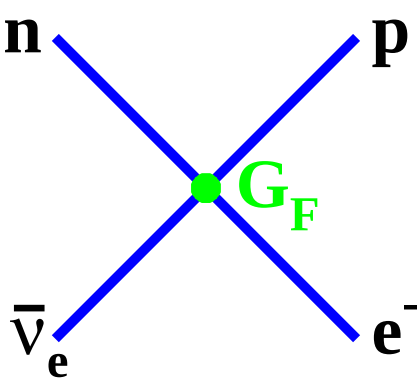


BOSON SELF-INTERACTIONS



★ **also interactions with PHOTONS (W-bosons are charged)**

Fermi Theory



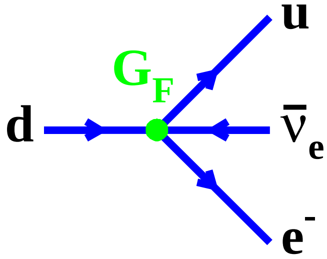
WEAK interaction taken to be a 4-fermion contact interaction

★ *i.e* no propagator

★ coupling strength G_F

★ G_F=1.166$\times 10^{-5}\ \mathrm{GeV}^{-2}$

Beta Decay in Fermi Theory



$\mathrm{n} \rightarrow \mathrm{p}\, \mathrm{e}^- \bar{\nu}_e$

Golden Rule :

$$1/\tau = \Gamma \quad = \quad 2\pi |\mathrm{M}_{fi}|^2 \rho(E)$$

$$\text{with} \quad \rho \quad = \quad \frac{dN}{dE}$$

Phase Space : 2-body vs. 3-body

★ TWO BODY FINAL STATE:

$$dN = \frac{E^2}{(2\pi)^3} d\Omega dE$$

(neglecting final state masses). Only consider one of the particles since the other fixed by (E,$\tilde{\mathrm{p}}$) conservation

★ THREE BODY FINAL STATE (*e.g* β-decay):

$$d^2N = \frac{E_\nu^2}{(2\pi)^3} d\Omega_\nu dE_\nu \frac{E_e^2}{(2\pi)^3} d\Omega_e dE_e$$

now necessary to consider phase space of two of the particles - the third is then given by (E,$\tilde{\mathrm{p}}$) conservation

In Nuclear β-decay the energy released in the nuclear transition, E_0, is shared between the electron, neutrino and the recoil kinetic energy of the nucleus:

$$E_0 = E_e + E_\nu + T_{\text{recoil}}$$

Since the nucleus is much more massive than the electron/neutrino:

$$E_0 \approx E_e + E_\nu$$

and the nuclear recoil ensures momentum conservation.

For a given electron energy E_e :

$$\begin{aligned} dE_\nu &= dE_0 \\ \frac{dN}{dE_0} &= \frac{dN}{dE_\nu} \\ &= \frac{E_\nu^2}{(2\pi)^3} d\Omega_\nu \frac{E_e^2}{(2\pi)^3} d\Omega_e dE_e \end{aligned}$$

Assuming isotropic decay distributions and integrating over $d\Omega_e d\Omega_\nu$ gives:

$$\begin{aligned} \frac{dN}{dE_0} &= (4\pi)^2 \frac{E_\nu^2}{(2\pi)^3} \frac{E_e^2}{(2\pi)^3} dE_e \\ &= \frac{E_\nu^2 E_e^2}{4\pi^4} dE_e \\ &= \frac{(E_0 - E_e)^2 E_e^2}{4\pi^4} dE_e \\ d\Gamma &= 2\pi |\mathrm{M}_{fi}|^2 \frac{(E_0 - E_e)^2 E_e^2}{4\pi^4} dE_e \\ \frac{d\Gamma}{dE_e} &= |\mathrm{M}_{fi}|^2 \frac{(E_0 - E_e)^2 E_e^2}{2\pi^3} \end{aligned}$$

In FERMI theory take:

$$|\mathrm{M}_{fi}|^2 = G_{\mathrm{F}}{}^2 \times f|M_{\text{nuclear}}|^2$$

where the nuclear matrix element $|M_{\text{nuclear}}|^2$ accounts for the overlap of the nuclear wave-functions, and f is the Coulomb correction.

Here assume $|M_{\text{nuclear}}|^2 = 1$ (super-allowed transition) and neglect f.

$$\Rightarrow \quad \frac{d\Gamma}{dE_e} = \frac{G_{\mathrm{F}}{}^2}{2\pi^3}(E_0 - E_e)^2 E_e^2$$

$$\Gamma = \frac{G_{\mathrm{F}}{}^2}{2\pi^3}\int_0^{E_0} (E_0 - E_e)^2 E_e^2 dE_e$$

$$\Gamma = \frac{G_{\mathrm{F}}{}^2}{2\pi^3}\left[\frac{E_0^5}{3} - 2\frac{E_0^5}{4} + \frac{E_0^5}{5}\right]$$

$$\Gamma = \frac{G_{\mathrm{F}}{}^2 E_0^5}{60\pi^3}$$

SARGENT RULE:

$$\boxed{\tau \propto E^{-5}}$$

★ e.g. see μ^- and τ^- decay

By studying lifetimes for nuclear beta decay (and applying necessary corrections, determine strength of weak coupling in FERMI theory:

$$G^{\beta} = 1.136 \pm 0.003 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Beta-Decay Spectrum

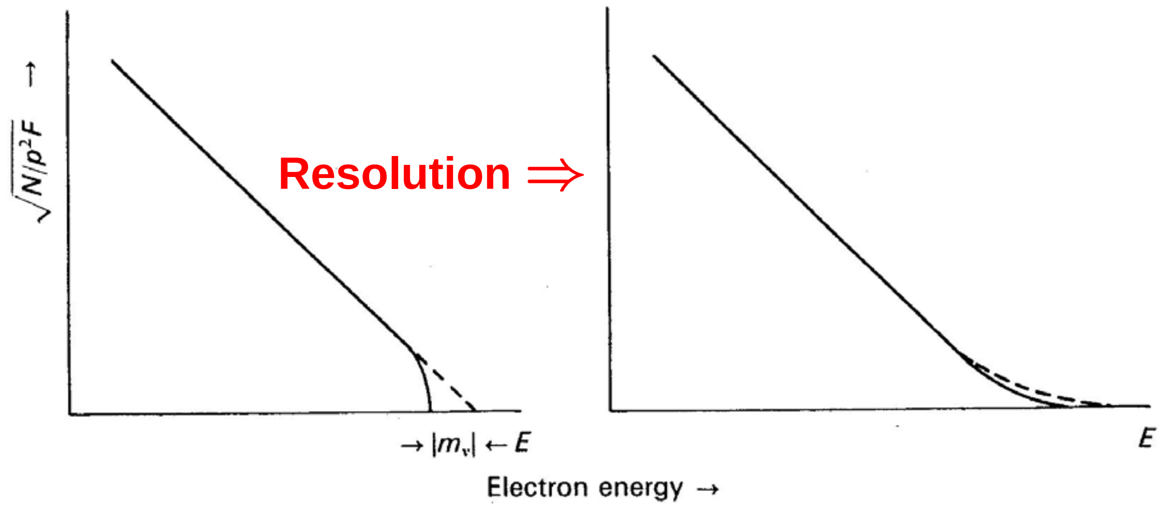
$$\frac{d\Gamma}{dE_e} = \frac{{G_F}^2}{2\pi^3}(E_0 - E_e)^2 E_e^2$$

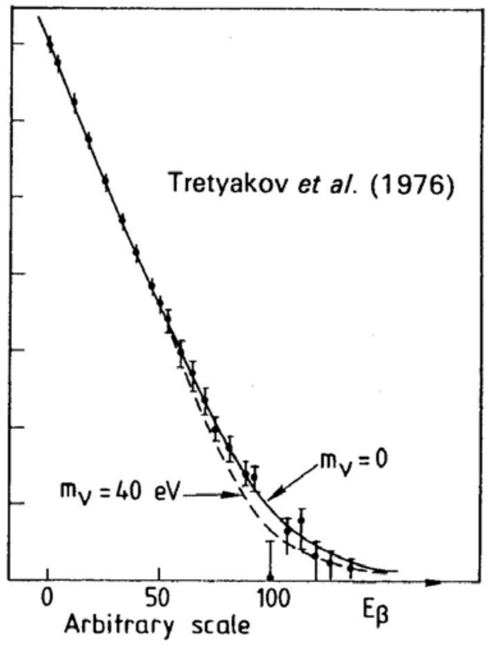
Plot of $\sqrt{\frac{d\Gamma}{dE_e}\frac{1}{E_e^2}}$ versus $(E_0 - E_e)$ (Kurie plot) is linear

$$\sqrt{\frac{d\Gamma}{dE_e}\frac{1}{E_e^2}} \propto (E_0 - E_e)$$

For a non-zero neutrino mass this is modified to

$$\frac{d\Gamma}{dE_e} = \frac{{G_F}^2}{2\pi^3}(E_0 - E_e)^2 E_e^2 \sqrt{1 - \left(\frac{m_\nu^2}{E_0 - E}\right)^2}$$





Most recent results (1999) Tritium $\beta-$decay:

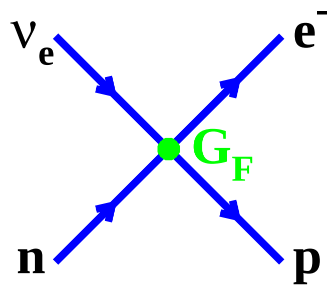
$$\boxed{m(\nu_e) < 3\,\mathrm{eV}}$$

<u>If</u> neutrinos have mass $m(\nu_e) \ll m(e)$.

Why so small ?

Neutrino Scattering in Fermi Theory (inverse β-decay)

$$\nu_e + n \rightarrow p + e^-$$



$$d\sigma = 2\pi |M_{fi}|^2 \frac{dN}{dE}$$
$$\sim 2\pi {G_F}^2 \frac{E_e^2}{(2\pi)^3} d\Omega$$
$$\sigma \sim {G_F}^2 s$$

where E_e is the energy of the e^- in the centre-of-mass system and $\sqrt{s}$ is the energy in the centre-of-mass system.

In the Laboratory frame: $s = 2E_\nu m_n$ (see Handout I)

$$\sigma(\nu_e n) \sim (E_\nu \text{ in MeV}) \times 10^{-43}\,\mathrm{cm}^2$$

★ Neutrinos only interact WEAKLY $\therefore$ have very small interaction cross-sections.

★ Here WEAK implies that you need approximately 50 light-years of water to stop a 1 MeV neutrino.

★ Communication via neutrino beams (á la Star Trek) non-trivial !

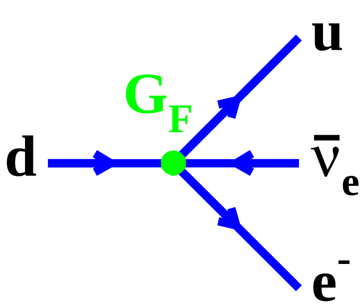
However, as $E_\nu \rightarrow \infty$ the cross-section $\sigma(\nu_\mu e^-)$ can become large. Violates maximum value allowed by conservation of probability at $\sqrt{s} = 740\ \mathrm{GeV}$ (UNITARITY LIMIT)

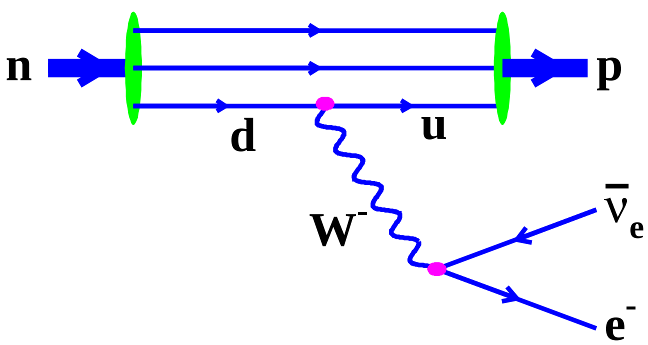
★ FERMI Theory breaks down at high energies

Weak Charged Current - $W^{\pm}$ Boson

- ★ Fermi theory breaks down at high energy.
- ★ True interaction described by exchange of charged W-bosons ($W^{\pm}$)
- ★ Fermi theory is the low energy ($q^2 \ll M_W{}^2$) EFFECTIVE theory of the WEAK interaction

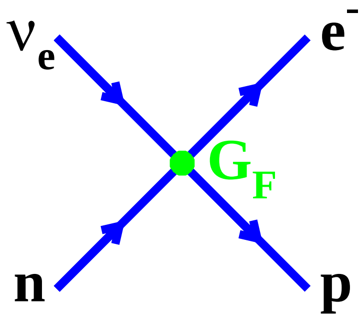
Beta-Decay:

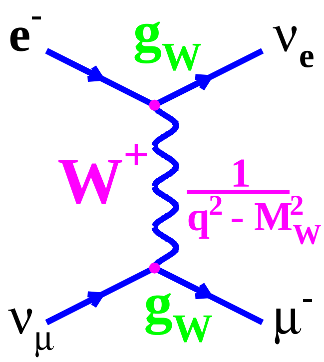




$$n \rightarrow p\, e^- \bar{\nu}_e$$

$\nu_\mu e^-$ Scattering:



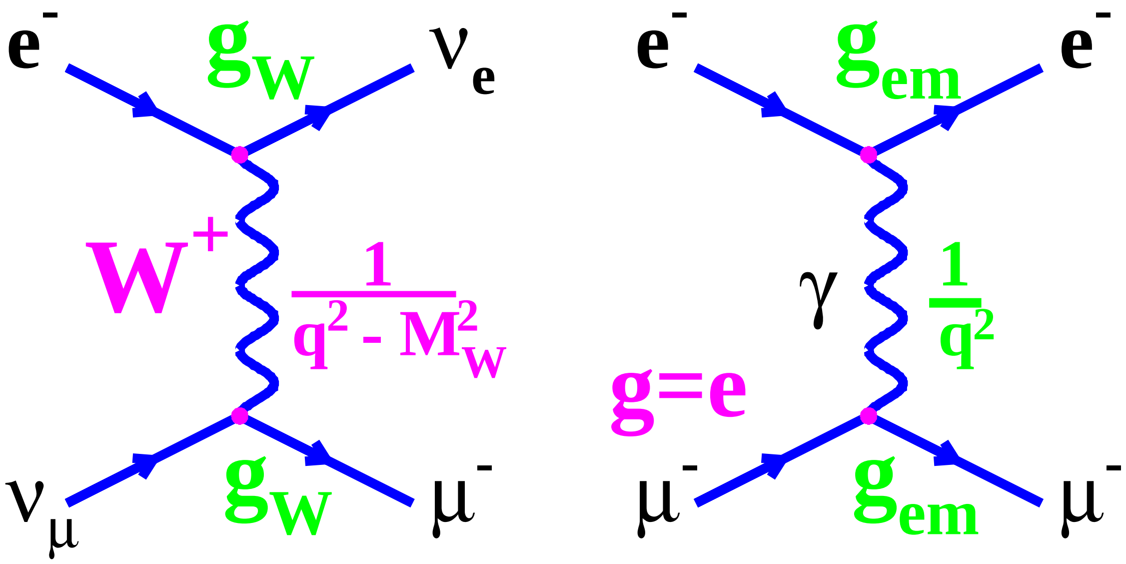


At low "energies" $q^2 << M_W{}^2$:

W-Boson propagator $\frac{1}{q^2 - M_W{}^2} \rightarrow \frac{1}{-M_W{}^2}$

Compare WEAK and QED interactions

WEAK INTERACTION **QED**



CHARGED CURRENT WEAK INTERACTION

★ For $q^2 \ll {M_W}^2$ propagator becomes $\frac{1}{{M_W}^2}$ - *i.e* appears as the POINT-LIKE interaction of FERMI theory.

★ Massive Propagator $\longrightarrow$ short range

$$M_W = 80.4 \pm 0.1 \text{ GeV}$$

Range $\approx \frac{1}{M_W}$ ~0.002 fm

★ Exchanged Boson carries electro-magnetic charge

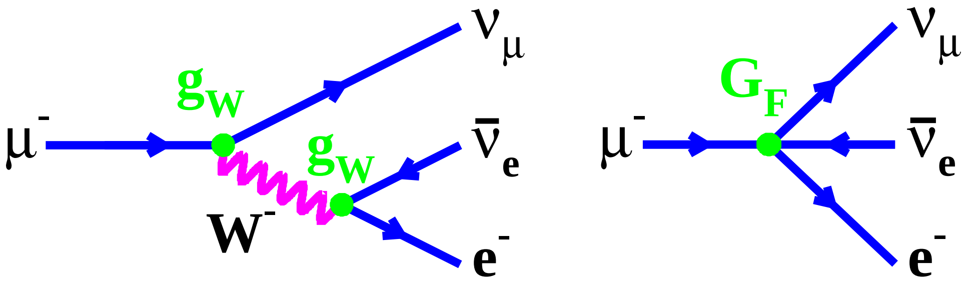
★ FLAVOUR CHANGING !

ONLY WEAK interaction changes flavour

★ Parity Violating !

ONLY WEAK interaction can violate parity conservation

COMPARE Fermi theory *c.f.* massive propagator



For $q^2 \ll M_W$ compare matrix elements:

$$\frac{g_W^2}{M_W^2} \quad \rightarrow \quad G_F$$

★ G_F is small because M_W is large.

The precise relationship is:

$$\frac{g_W^2}{8M_W^2} \quad \rightarrow \quad \frac{G_F}{\sqrt{2}}$$

The numerical factors are partly of historical origin (*e.g.* see Perkins 4^{th} Edition, page 210).

$M_W = 80.4\,\mathrm{GeV}$ and $G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$

$$\Rightarrow \quad g_W \quad = \quad 0.65$$
$$\therefore \quad \alpha_W \quad = \quad \frac{g_W^2}{4\pi} \approx \frac{1}{30}$$

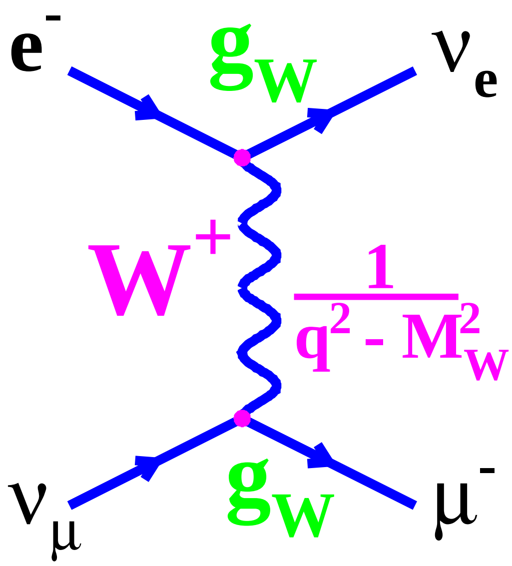
The intrinsic strength of the WEAK interaction is greater than that of the electro-magnetic interaction. At low energies (low q^2) it appears weak due to the massive propagator.

★ $\alpha_S \approx 0.2$, $\alpha_W \approx 0.03$, $\alpha_{EM} \approx 0.01$

★ suggestive of UNIFICATION of the forces

Neutrino Scattering with a Massive W Boson

Replace contact interaction by massive boson exchange diagram:



$$\frac{d\sigma}{dq^2} = \frac{1}{32\pi}\frac{g_w^4}{(q^2 - M_W{}^2)^2}$$

with $|q| = 2E\sin\frac{\theta}{2}$

where θ is the scattering angle.

(e.g. similar to Handout I p.36)

Integrate to give:

$$\sigma = \frac{G_F{}^2 s}{\pi} \qquad s \ll M_W{}^2$$

$$\sigma = \frac{G_F{}^2 M_W{}^2}{\pi} \qquad s \gg M_W{}^2$$

Total cross section now well behaved at high energies.

Parity Violation in Beta Decay

Revision : Nuclear Physics

Under Parity $\hat{\mathrm{P}}$:

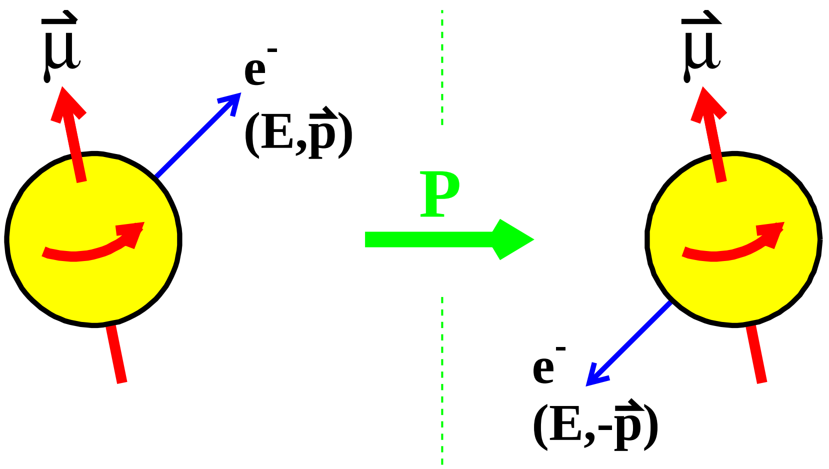
$$\begin{aligned} \tilde{\mathrm{r}} &\rightarrow -\tilde{\mathrm{r}} \\ \tilde{\mathrm{p}} \propto \nabla &\rightarrow -\tilde{\mathrm{p}} \\ \tilde{\mathrm{L}} = \tilde{\mathrm{r}} \times \tilde{\mathrm{p}} &\rightarrow \tilde{\mathrm{L}} \\ \tilde{\mu} &\rightarrow \tilde{\mu} \end{aligned}$$

$\hat{\mathrm{P}}$: Axial vectors *e.g.* $\tilde{\mathrm{L}}$, $\tilde{\mu}$ do not change sign

<u>EXPERIMENT:</u> Align ^{60}Co nuclei at low temperatures with $\tilde{\mathrm{B}}$ field

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}\, \mathrm{e}^- \,\overline{\nu}_\mathrm{e}$$

Observe angular distribution of e^- relative to $\tilde{\mathrm{B}}$.



If parity conserved expect equal numbers of e^- parallel and anti-parallel to $\tilde{\mathrm{B}}$.

Experiment (C.S. Wu 1956) showed clear asymmetry $\Rightarrow$ PARITY VIOLATION in WEAK interactions

Origin of Parity Violation

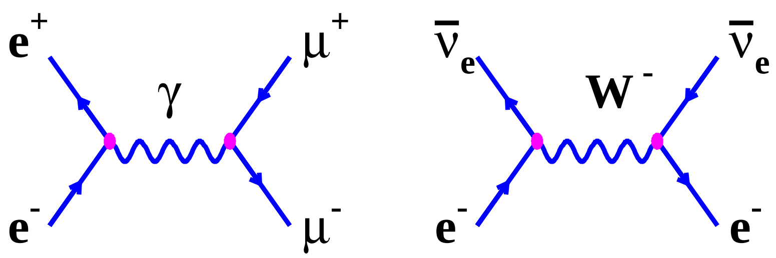
In the ultra-relativistic (massless) limit **only**

- ★ LEFT-HANDED PARTICLES and
- ★ RIGHT-HANDED ANTI-PARTICLES.

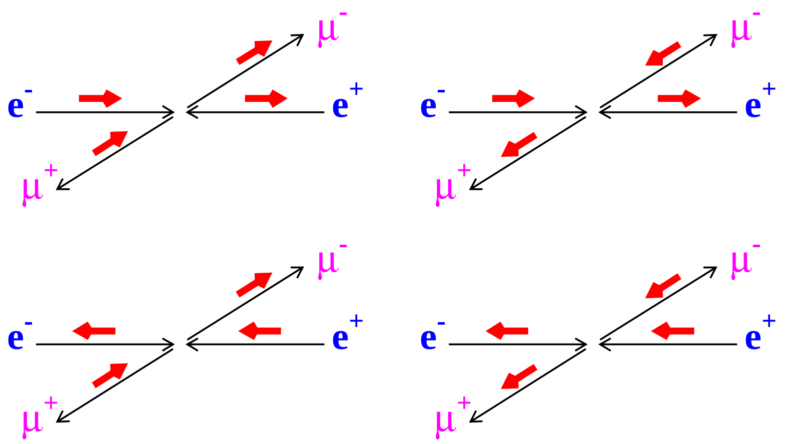
participate in the WEAK (charged current) interaction.

For massive fermions the weak interaction couples preferentially to LEFT-HANDED particles and RIGHT-HANDED anti-particles.

Compare QED and WEAK interaction.

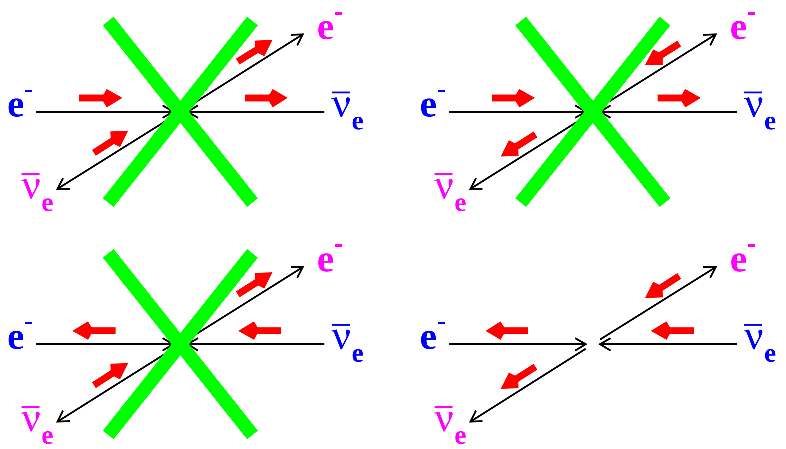


QED : $e^+e^- \rightarrow \mu^+\mu^-$



From 16 possible SPIN assignments only 4 give non-zero contributions to cross section

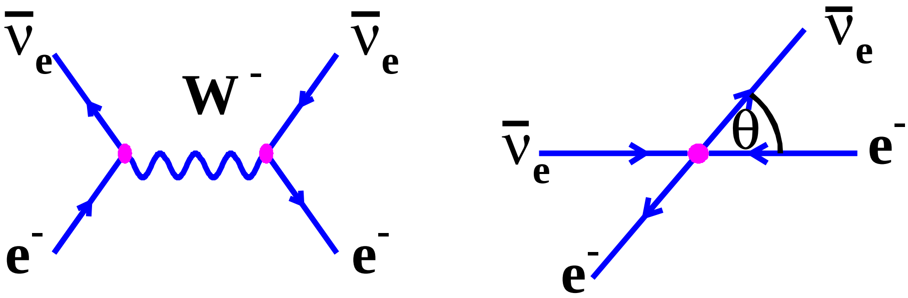
WEAK INTERACTION : $\overline{\nu}_e e^- \rightarrow \overline{\nu}_e e^-$



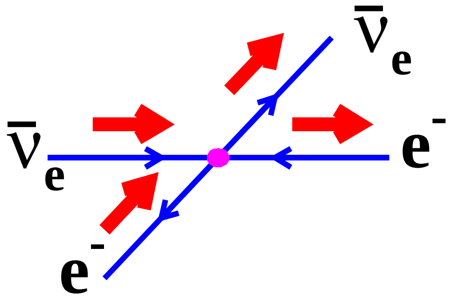
From 16 possible SPIN assignments only 1 gives a non-zero contribution to cross section.

$L\overline{R} \rightarrow L\overline{R}$

<u>EXAMPLE</u> $\overline{\nu}_e e^- \to \overline{\nu}_e e^-$ **scattering in the centre-of-mass frame** $(s = E_e + E_\nu = 2E_e)$



In massless limit - only one Helicity state contributes:



$$\frac{d\sigma}{d\Omega} = 2\pi|M_{fi}|^2\frac{E_e^2}{(2\pi)^3}$$
$$= \frac{1}{16\pi^3}|M_{fi}|^2 s$$

where s **is centre-of-mass energy**

$$M_{fi} = \left(\frac{g_W}{\sqrt{2}}\right)^2\frac{1}{q^2 - M_W^2}\cos^2\frac{\theta}{2}$$

For $q^2 \ll M_W^2$

$$|M_{fi}| = \frac{2G_F}{\sqrt{2}}(1+\cos\theta)$$
$$\frac{d\sigma}{d\Omega} = \frac{1}{8\pi^2}{G_F}^2 s(1+\cos\theta)^2$$
$$\sigma = \frac{{G_F}^2 s}{3\pi}$$

Parity Violation

The WEAK interaction treats LH and RH states differently and therefore can violate PARITY (*i.e.* the interaction Hamiltonian does not commute with $\hat{\mathrm{P}}$)

Parity ALWAYS conserved in STRONG/EM interactions

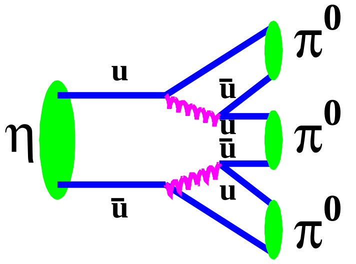
$$P = \prod_i P_i \prod_{i>j} (-1)^{L_{ij}}$$

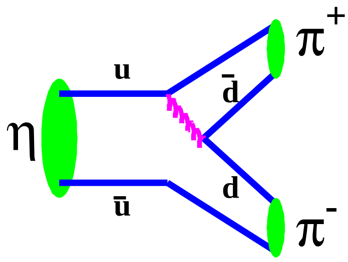
where P_i is the intrinsic parity of the particle i and L_{ij} is the orbital angular momentum between particles i and j.

Taking $L_{ij} = 0$

$\eta \rightarrow \pi^0\pi^0\pi^0$ | J^P: 0^- → $0^-\ 0^-\ 0^-$ | P: -1 → -1

$\eta \rightarrow \pi^+\pi^-$ | J^P: 0^- → $0^-\ 0^-$ | P: -1 → $+1$



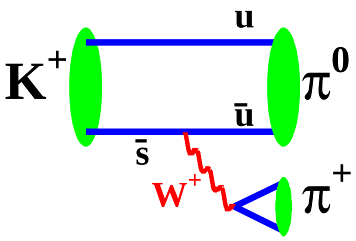


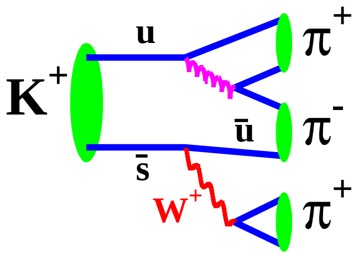
BR = 32 % | **BR < 0.1 %**

Parity is usually violated in WEAK interactions

$K^+ \rightarrow \pi^+\pi^0$ | J^P: 0^- → $0^-\ 0^-$ | P: -1 → $+1$

$K^+ \rightarrow \pi^+\pi^-\pi^+$ | J^P: 0^- → $0^-\ 0^-\ 0^-$ | P: -1 → -1





BR = 21 % | **BR = 6 %**

but NOT ALWAYS !

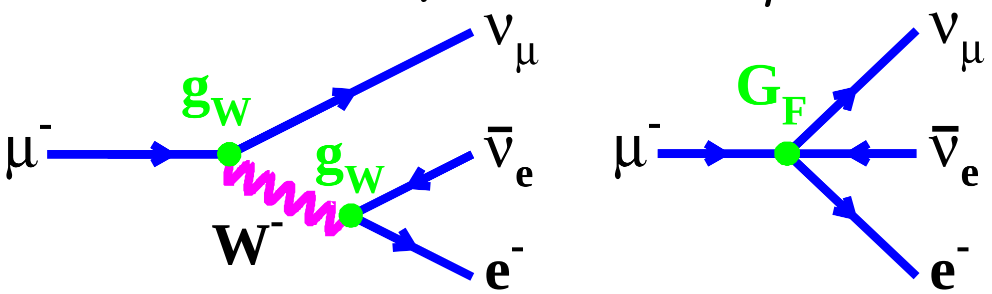
Weak Leptonic Decays

★ Muons are fundamental leptons ($m_\mu \approx 206 m_e$).

★ Electro-magnetic decay $\mu^- \to e^- \gamma$ IS NOT observed; the EM interaction does not change flavour.

★ Only the WEAK charged current changes flavour.

★ Muons decay weakly : $\mu^- \to e^- \overline{\nu}_e \nu_\mu$



As $m_\mu^2 \ll M_W{}^2 \Rightarrow$ can use FERMI theory to calculate decay width (analogous to β decay).

FERMI theory gives decay width proportional to m_μ^5 (Sargent Rule):

However more complicated phase space integration (previously neglected kinetic energy of recoiling nucleus)

gives $$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F{}^2}{192\pi^3} m_\mu^5$$

★ Muon mass and lifetime known with high precision.

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}\ \mathrm{s}$$

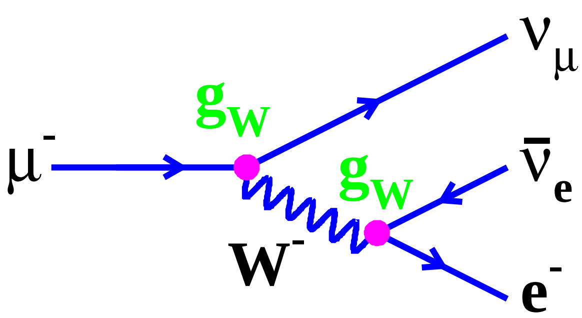
★ Use muon decay to fix strength of WEAK interaction G_F

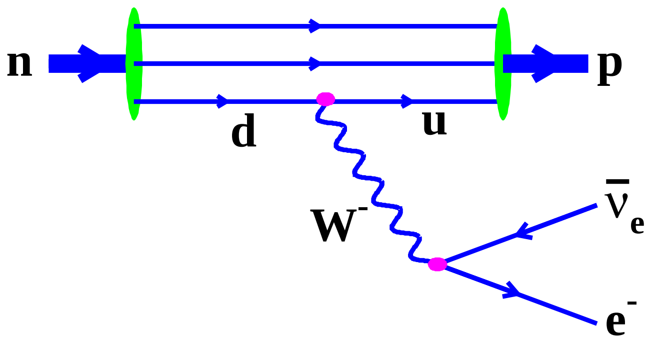
$$G_F = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

★ G_F is one of the best determined fundamental quantities in particle physics.

Universality of Weak Coupling

Can compare G_F measured from μ^--decay with that obtained from β-decay





From muon decay measure:

$$G^{\mu} = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

From β-decay measure:

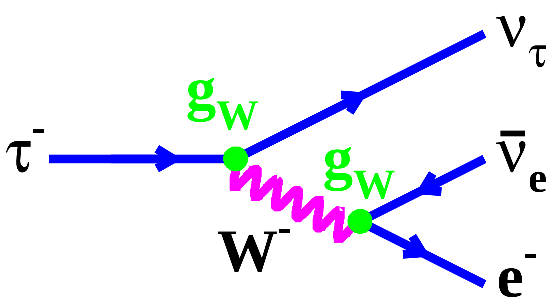
$$G^{\beta} = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Taking ratio gives

$$\frac{G^{\beta}}{G^{\mu}} = 0.974 \pm 0.003$$

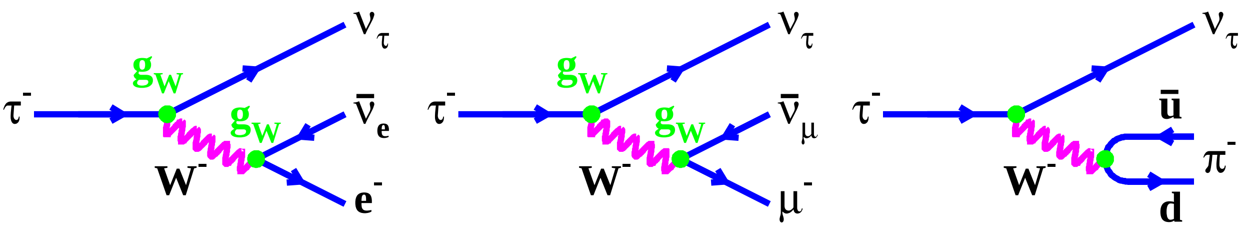
Conclude that the strength of the weak interaction is almost the same for muons/electrons as for up/down quarks and we'll shortly come back to the origin of this difference ($\cos\theta_c$)

Can also test universality of the WEAK interaction in τ-decays, *e.g.*



Tau Decays

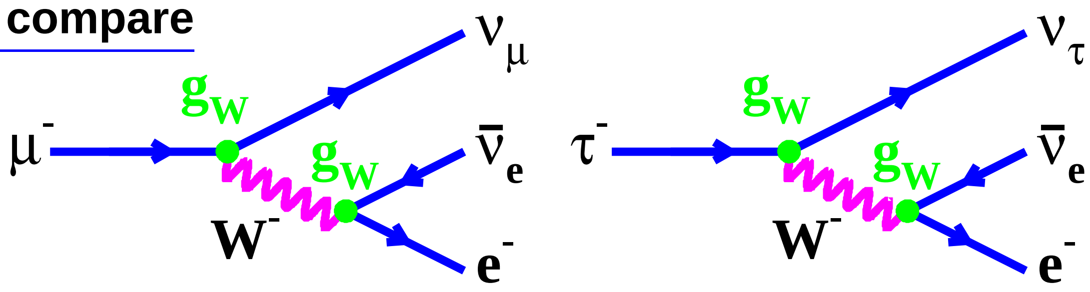
The τ mass is relatively large, $m_\tau = 1.777\,\mathrm{GeV}$,
and as $m_\tau > \{m_\mu, m_\pi, m_\rho, ...\}$
there a number of possible tau decay modes, *e.g.*



Tau Branching Fractions:

- ★ $\tau^- \rightarrow e^-\overline{\nu}_e\nu_\tau$ $(17.8 \pm 0.1\,\%)$
- ★ $\tau^- \rightarrow \mu^-\overline{\nu}_\mu\nu_\tau$ $(17.3 \pm 0.1\,\%)$
- ★ $\tau^- \rightarrow$ hadrons $(64.7 \pm 0.2\,\%)$

First compare



$$\frac{1}{\tau_\mu} = \Gamma_{\mu\rightarrow e} = \frac{{G_F}^2}{192\pi^3}m_\mu^5$$

$$\frac{1}{\tau_\tau} = \frac{1}{Br(\tau \rightarrow e)}\Gamma_{\tau\rightarrow e} = \frac{1}{0.178}\frac{{G_F}^2}{192\pi^3}m_\tau^5$$

If universal strength of WEAK interaction expect

$$\frac{\tau_\tau}{\tau_\mu} = 0.178\frac{m_\mu^5}{m_\tau^5}$$

m_μ, m_τ, τ_μ are all precisely measured

Using: $m_\mu = 105.658\ \mathrm{MeV}$

$$m_\tau = (1777.0 \pm 0.3)\ \mathrm{MeV}$$

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}\ \mathrm{s}$$

gives a PREDICTION of

$$\tau_\tau = 2.91 \pm 0.01 \times 10^{-13}\ \mathrm{s}$$

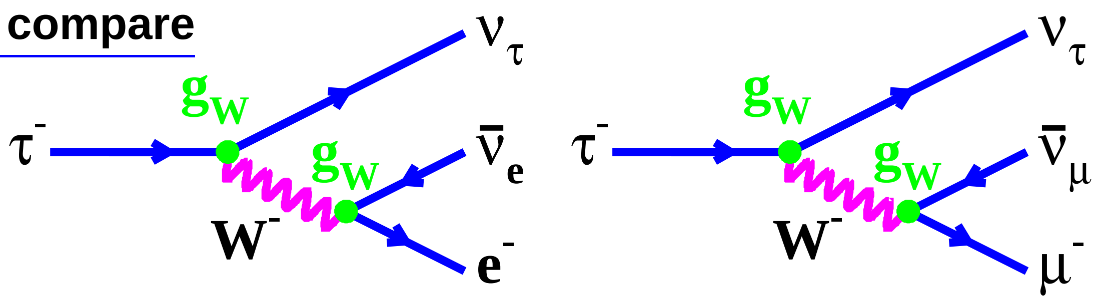
compare to MEASURED VALUE:

$$\tau_\tau = 2.91 \pm 0.01 \times 10^{-13}\ \mathrm{s}$$

Consistent with the predicted value, *i.e.*

★ Same WEAK CC coupling for μ and τ.

Also compare



IF same couplings expect:

$$\frac{Br(\tau^- \rightarrow \mu^- \overline{\nu}_\mu \nu_\tau)}{Br(\tau^- \rightarrow e^- \overline{\nu}_e \nu_\tau)} = 0.9726$$

(the small difference is due to the slight reduction in phase space due to the non-negligible muon mass)

The observed ratio

$$0.974 \pm 0.005$$

is consistent with the prediction 0.9726

★ Same WEAK CHARGED CURRENT coupling for e, μ and $\tau \rightarrow$ LEPTON UNIVERSALITY

(see Question 9 on the problem sheet)

W Leptonic Couplings

Standard Model W Boson Couplings

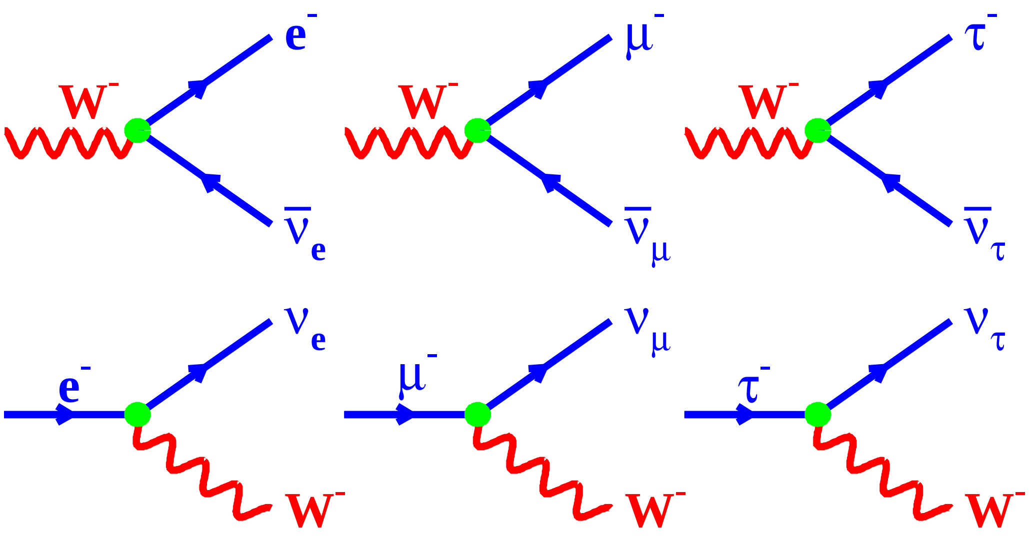
★ In the Standard Model the 'charge' of the WEAK interactions is called WEAK ISOSPIN.

★ Leptons are represented in Doublets

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

★ W-bosons only 'couple' particles within a doublet.

★ *e.g.* no $We^-\nu_\mu$ coupling.

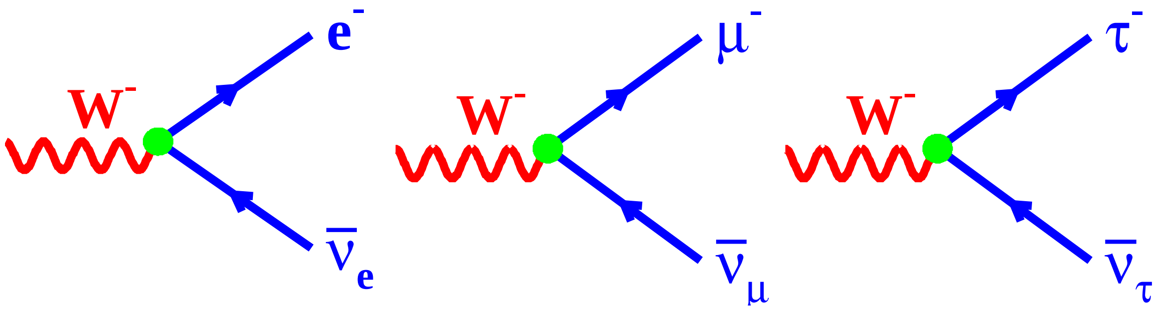


★ UNIVERSAL COUPLING STRENGTH

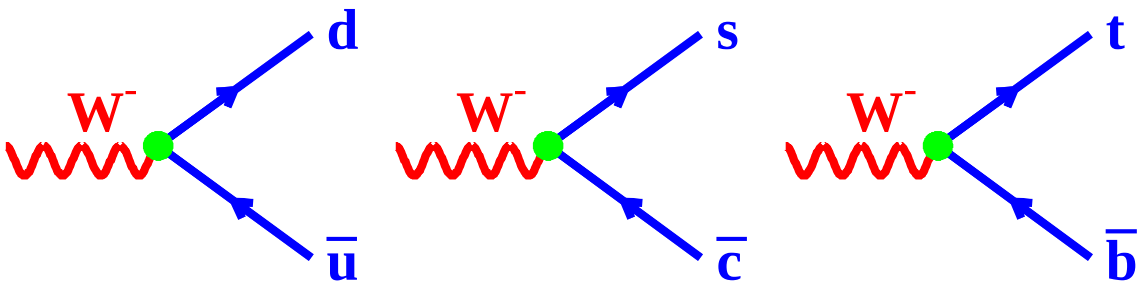
$$\frac{g_w}{\sqrt{2}}$$

Weak Interactions of Quarks

In the Standard Model, the leptonic weak couplings take place within generation,



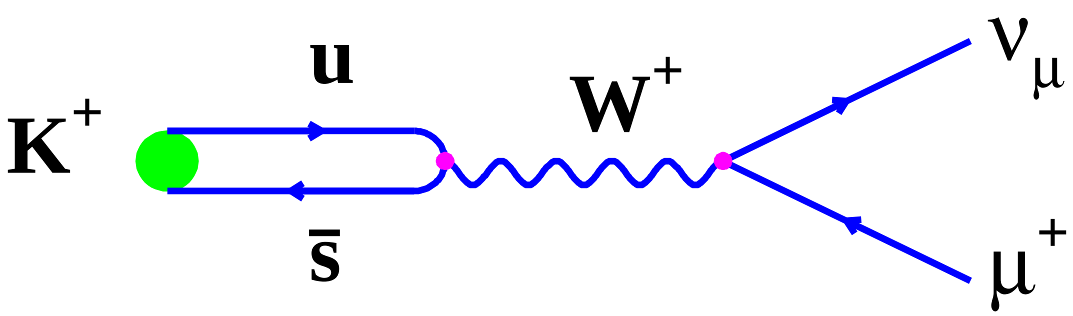
Natural to expect same Pattern for QUARKS *i.e.*



Unfortunately its not that simple !

Example

The decay $K^+(u\bar{s}) \to \mu^+\nu_\mu$ suggests a $W^+u\bar{s}$ coupling



Cabibbo Mixing Angle

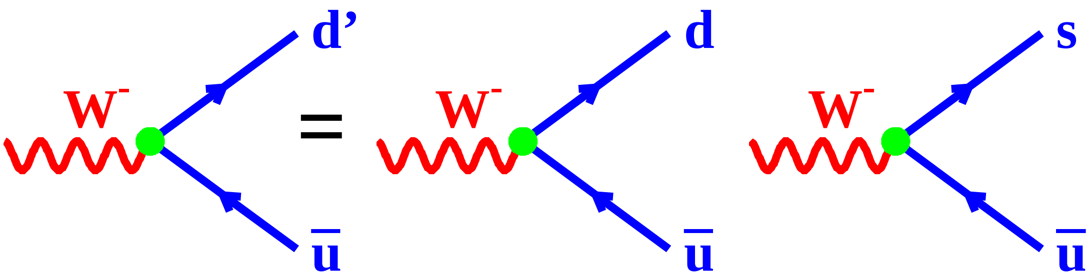
Four-Flavour Quark Mixing

★ the states which take part in the WEAK interaction are ORTHOGONAL combinations of the states of definite flavour (d,s)

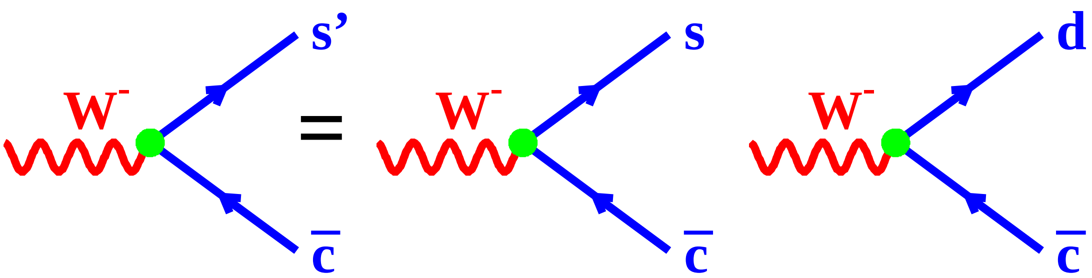
★ For 4-flavours, $\{\, d, u, s$ and $c\,\}$, the mixing can be described by a single parameter: the CABIBBO ANGLE θ_c

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Couplings become:



$\cos\theta_c\, d + \sin\theta_c\, s$ $\quad g_W \cos\theta_c$ $\quad g_W \sin\theta_c$



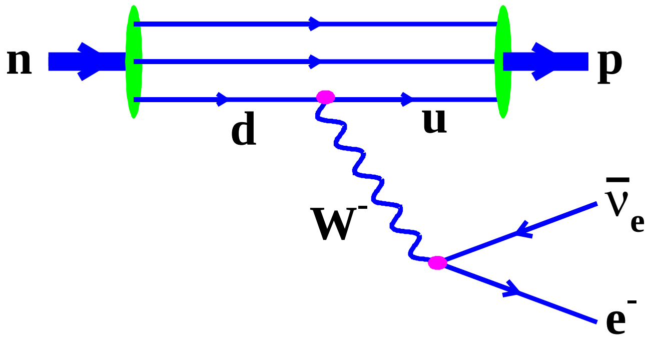
$\cos\theta_c\, s - \sin\theta_c\, d$ $\quad g_W \cos\theta_c$ $\quad -g_W \sin\theta_c$

EXPERIMENTALLY:

$$\boxed{\theta_c = 13^\circ}$$

EXAMPLE: Nuclear Beta Decay

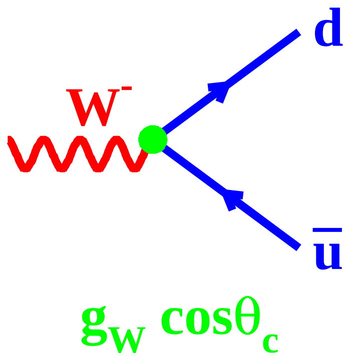
Recall:

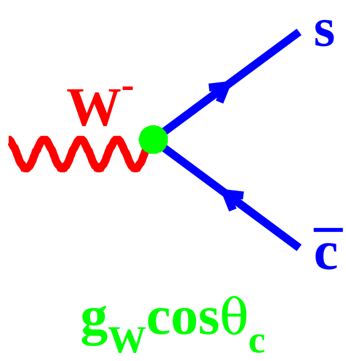


$$G_\mu = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$
$$G_\beta = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

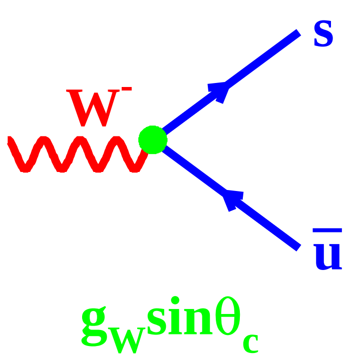
- ★ Strength of ud coupling $\propto g_w \cos\theta_c$
- ★ $(G_\beta)^2 \propto |M|^2 \propto \cos^2\theta_c$
- ★ Hence expect $G_\beta = \cos\theta_c G_\mu$
- ★ It works, $1.16632 \times \cos 13^\circ = 1.136$

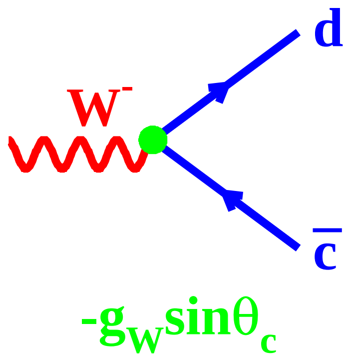
Cabibbo Favoured : $|M|^2 \propto \cos^2\theta_c$



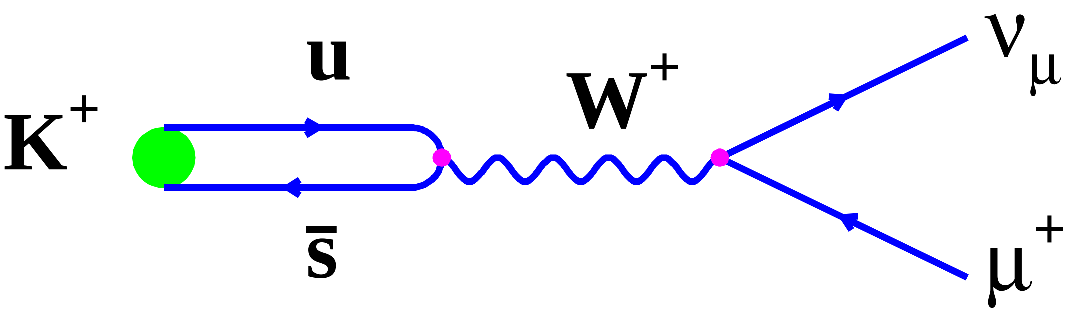


Cabibbo Suppressed : $|M|^2 \propto \sin^2\theta_c$





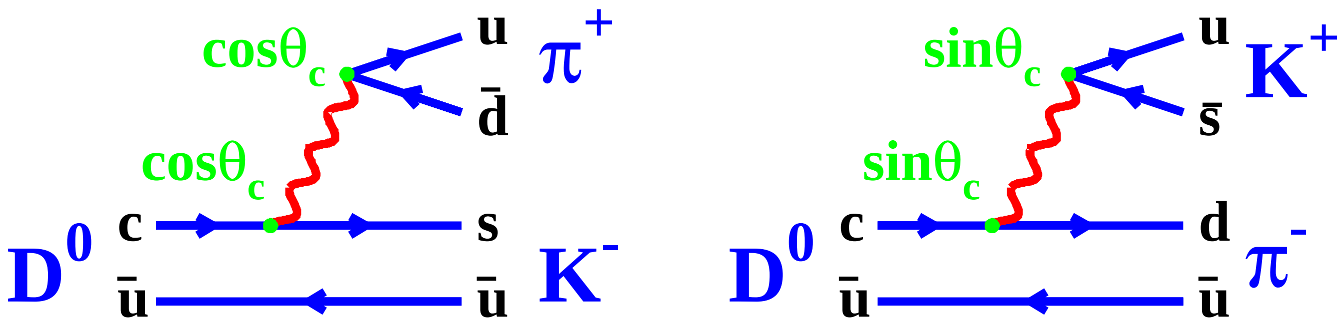
EXAMPLE: $K^+ \to \mu^+\nu_\mu$



$u\bar{s}$ coupling $\Rightarrow$ Cabibbo suppressed

$|M|^2 \propto \sin\theta_c^2$

EXAMPLE: $D^0 \to K^-\pi^+$, $D^0 \to K^+\pi^-$



Expect

$$\frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} = \frac{\sin^4\theta_c}{\cos^4\theta_c}$$

$$\approx \quad 0.0028$$

Measure $\quad 0.0038 \pm 0.0008$

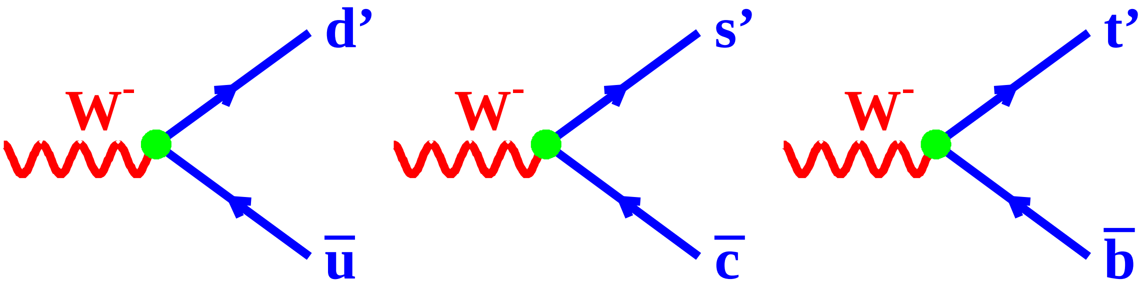
$D^0 \to K^+\pi^-$ is DOUBLY Cabibbo suppressed

(see Question 8 on the problem sheet)

CKM Matrix

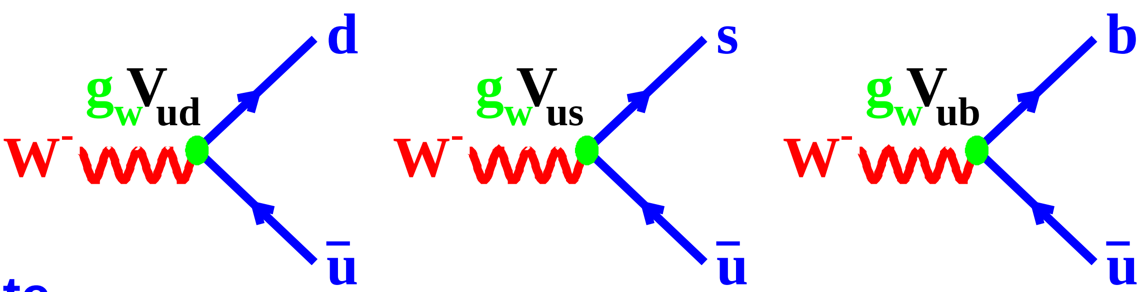
Cabibbo-Kobayashi-Maskawa Matrix

Extend to 3 generations



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Giving couplings



Note

$$V_{ckm} \approx \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0.01 \\ -\sin\theta_c & \cos\theta_c & 0.05 \\ 0.01 & -0.05 & 1 \end{pmatrix}$$

sometimes written

$$V_{ckm} \approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

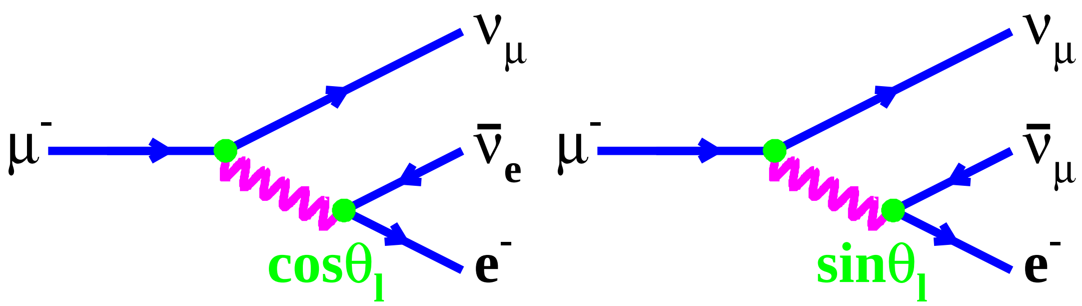
with $\lambda = \sin\theta_c$

(see Question 10 on the problem sheet)

Lepton Mixing Matrix ?

Natural to ask if there is an equivalent of the CKM Matrix for leptons.

HYPOTHETICAL EXAMPLE:



The neutrinos are unobserved, (*i.e.* don't distinguish the different neutrino final states). Consequently the amplitude for $\mu^- \rightarrow e^- \nu \overline{\nu}$

$$|M|^2 \quad \propto \quad g_w^2 (\cos^2\theta_l + \sin^2\theta_l)$$

★ In the quark sector, mass differences between quarks (and the hadrons they form) allow us to distinguish the different final states

See Handout VIII for the evidence that there is MIXING in the lepton sector

Summary

WEAK INTERACTION (CHARGED-CURRENT)

★ Parity violated due to the HELICITY structure of the interaction

★ Force mediated by massive W-bosons, $M_{\mathbf{W}} = 80.4$ GeV

★ Intrinsically stronger than EM interaction

★ Universal coupling to quarks and leptons

★ Quarks take part in the interaction as mixtures of the flavour eigenstates

★ $G_{\mathbf{F}}$=$(1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$ from muon decay

ELECTROWEAK UNIFICATION - next handout

★ Neutral Current WEAK interaction - Z^0

★ Unification of WEAK and EM forces

APPENDIX: VECTOR-AXIAL VECTOR (V—A)

NON-EXAMINABLE

In the DIRAC equation the WEAK interaction vertex has the form VECTOR — AXIAL-VECTOR $\gamma^\mu(1-\gamma^5)$

Consider Dirac spinors for a particle traveling along the z-axis

$$u_R = N \begin{pmatrix} 1 \\ 0 \\ \frac{p}{(E+m)} \\ 0 \end{pmatrix} \qquad u_L = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p}{(E+m)} \end{pmatrix}$$

The WEAK interaction matrix element looks like

$$\langle \bar{u}_R | \gamma^\mu (1-\gamma^5) | u_L \rangle$$

it has the form VECTOR (γ^μ) minus AXIAL-VECTOR $\gamma^\mu\gamma^5$

In matrix form:

$$\begin{aligned} 1-\gamma^5 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \end{aligned}$$

Consider the effect of the interaction on LH and RH spinors:

$$(1-\gamma^5)u_R = N\begin{pmatrix} 1-\frac{p}{(E+m)} \\ 0 \\ -1+\frac{p}{(E+m)} \\ 0 \end{pmatrix}$$

$$(1-\gamma^5)u_L = N\begin{pmatrix} 0 \\ 1+\frac{p}{(E+m)} \\ 0 \\ -1-\frac{p}{(E+m)} \end{pmatrix}$$

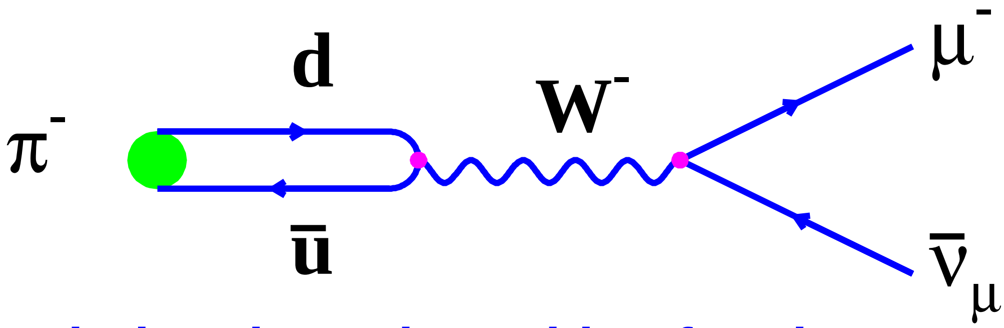
Massless limit, $m \to 0, p \to E$:

$$(1-\gamma^5)u_R = 0$$

- ★ For massless particles, the form of the interaction projects out LH particle states, i.e. only LH-particles take part in the WEAK interaction.
- ★ For massive particles, the form of the interaction preferentially projects out LH particles.

IF neutrinos were massless (which is not quite the case), the WEAK couplings of RH neutrinos and LH anti-neutrinos would be zero, and if the ν_R and $\overline{\nu}_L$ state exist they would only experience the gravitational interaction !

EXAMPLE: $\pi^{\pm}$ Decay



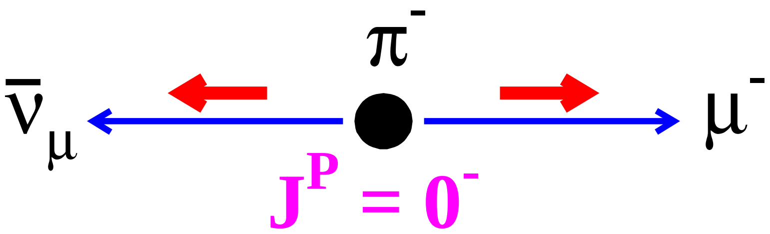
Charged Pion decay branching fractions:

★ $\pi^- \to \mu^- \overline{\nu}_\mu$ $\quad$ $99.9877\ \%$

★ $\pi^- \to e^- \overline{\nu}_e$ $\quad$ $0.0123\ \%$

Naïvely might expect slightly larger branching fraction for $\pi^- \to e^- \overline{\nu}_e$ due to phase space !

Consider Spin/Helicity



★ **Conservation of angular momentum $\Rightarrow$ muon and neutrino spins in opposite directions.**
SAME HELICITY

★ **Neutrinos massless, $\therefore$ only RH anti-neutrino takes part in WEAK interaction**

★ **therefore μ^- is also right-handed**

★ **IF massless, *e.g.* $m_\mu = 0$, the WEAK Matrix element would be exactly zero**

$$(1-\gamma^5)u_R \quad = \quad N\begin{pmatrix} 1\text{-}\frac{p}{(E+m)} \\ 0 \\ \text{-}1\text{+}\frac{p}{(E+m)} \\ 0 \end{pmatrix}$$

"Wrong-Handed" ME (zero for $m = 0$)

$$M \quad \propto \quad f_{wrong} = \frac{1}{2}\left(1 - \frac{p}{E+m}\right)$$

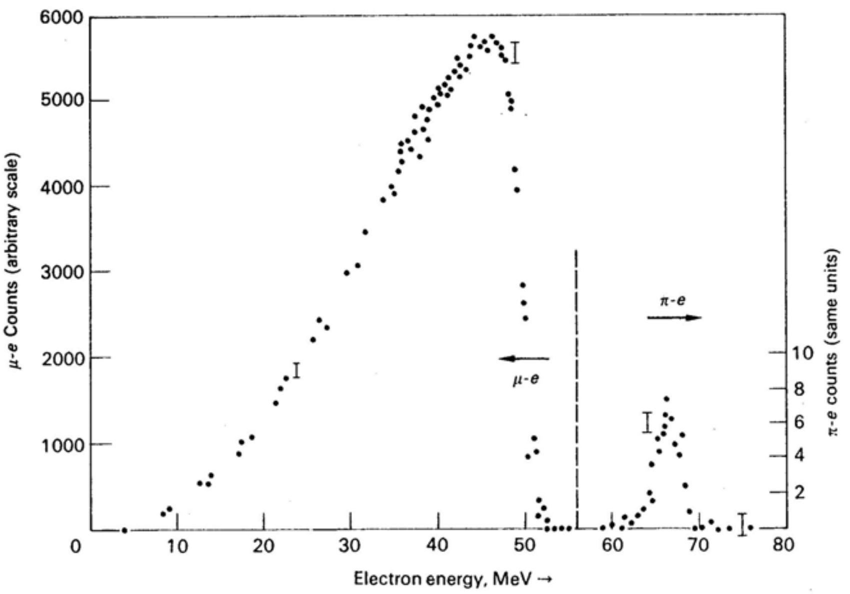
$\pi^- \rightarrow$	p_{lept}	E_{lept}	f_{wrong}
$\mu^-\overline{\nu}_\mu$	30 MeV	110 MeV	0.43
$e^-\overline{\nu}_e$	70 MeV	70 MeV	0.0035

★ $\pi^- \rightarrow \mu^-\overline{\nu}_\mu$ **is non-relativistic**

★ **Decay $\pi^- \rightarrow e^-\overline{\nu}_e$ suppressed relative to $\pi^- \rightarrow \mu^-\overline{\nu}_\mu$: $(\frac{0.0035}{0.43})^2 \approx 6 \times 10^{-5}$**

★ **Once phase-space taken into account:**

$$\frac{\Gamma(\pi^- \rightarrow e^-\overline{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^-\overline{\nu}_\mu)} = 1.23 \times 10^{-4}$$



Positron decay spectrum from π^+ decays.

Large Peak

$$\pi^+ \rightarrow \mu^+\nu_\mu$$
$$\mu^+ \rightarrow e^+\nu_e\overline{\nu}_\mu$$

Small Peak

$$\pi^+ \rightarrow e^+\nu_e$$