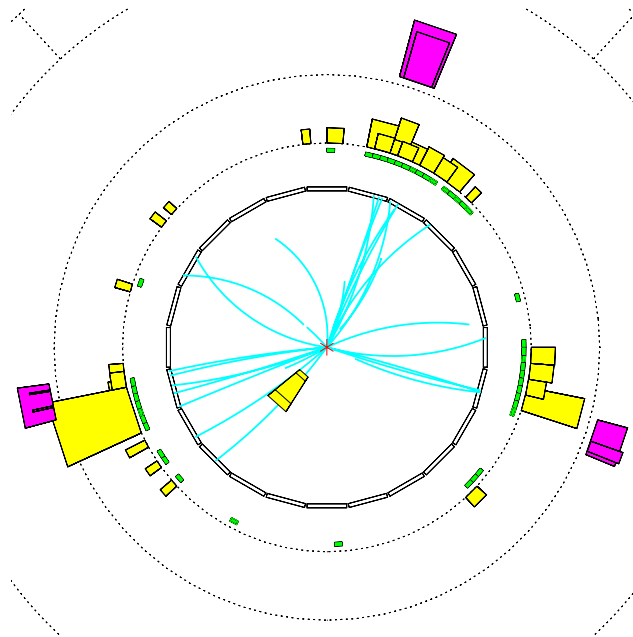
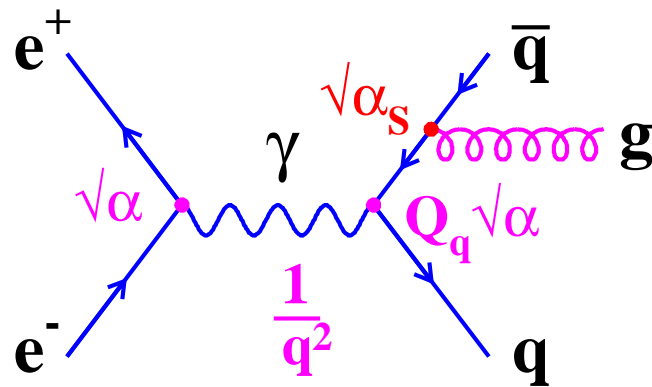


Particle Physics

Dr M.A. Thomson



Part II, Lent Term 2004
HANDOUT III

QCD

QUANTUM ELECTRODYNAMICS: is the quantum theory of the electromagnetic interaction.

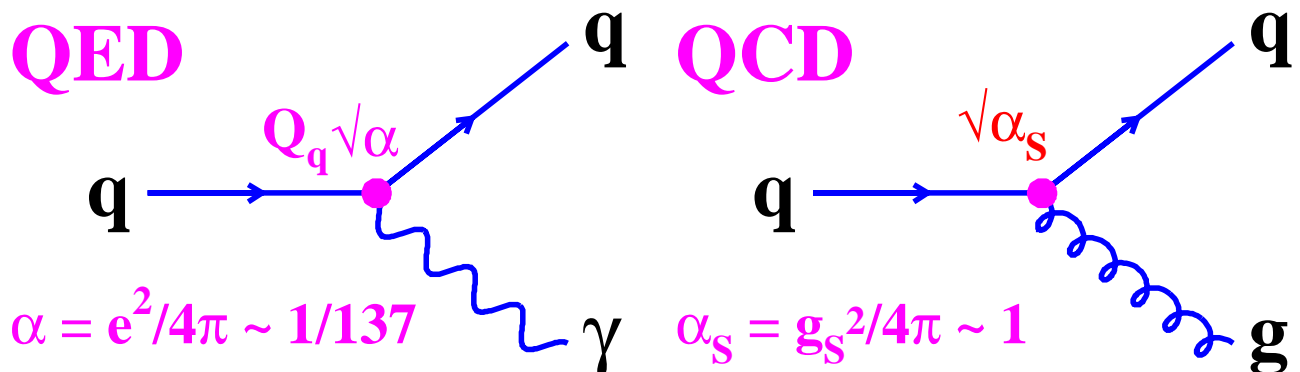
- ★ mediated by massless photons
- ★ photon couples to electric charge, e
- ★ Strength of interaction : $\langle \psi_f | \hat{H} | \psi_i \rangle \propto \sqrt{\alpha}$.

$$\alpha = \frac{e^2}{4\pi}$$

QUANTUM CHROMO-DYNAMICS: is the quantum theory of the strong interaction.

- ★ mediated by massless gluons, i.e. $1/q^2$ propagator
- ★ gluon couples to “strong” charge
- ★ Only quarks have non-zero “strong” charge, therefore only quarks feel strong interaction

Basic QCD interaction looks like a stronger version of QED, $\alpha_S > \alpha_{EM}$



(subscript em is sometimes used to distinguish the α_{em} of electromagnetism from α_S).

COLOUR

In QED:

- ★ Charge of QED is electric charge.
- ★ Electric charge - conserved quantum number.

In QCD:

- ★ Charge of QCD is called “COLOUR”
- ★ COLOUR is a conserved quantum number with 3 VALUES labelled “red”, “green” and “blue”

Quarks carry “COLOUR” $r \quad g \quad b$
 Anti-quarks carry “ANTI-COLOUR” $\bar{r} \quad \bar{g} \quad \bar{b}$

Leptons, γ , W^\pm , Z^0 DO NOT carry colour, i.e.
 “have colour charge zero” \rightarrow DO NOT participate
 in STRONG interaction.

Note: Colour is just a label for states in a
 non-examinable SU(3) representation

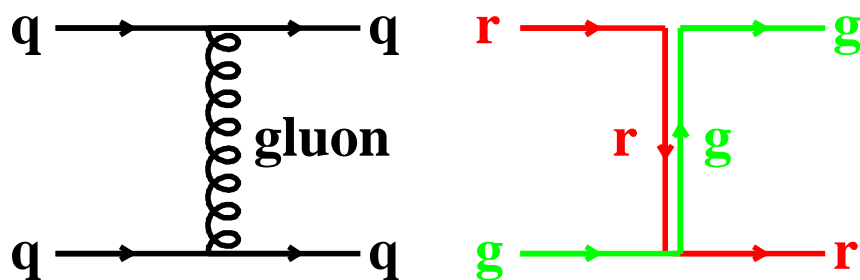
$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

GLUONS

In QCD:

★ Gluons are **MASSLESS** spin-1 bosons

Consider a **red quark** scattering off a **green quark**.
Colour is exchanged but always conserved.



UNLIKE QED:

★ Gluons carry the charge of the interaction.

★ Gluons come in different colours.

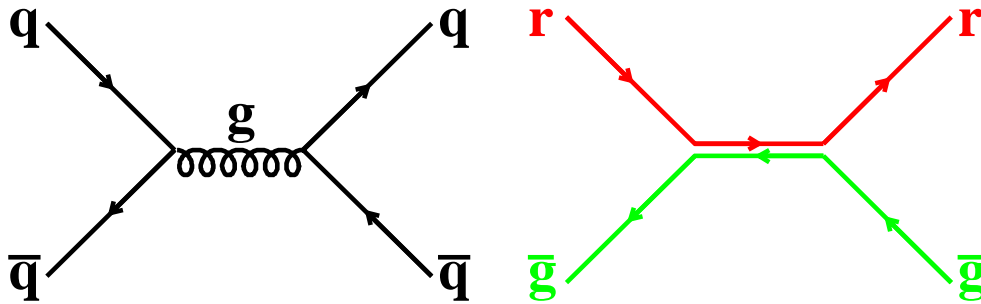
Expect 9 gluons (3 colours \times 3 anti-colours)

$$r\bar{b}, r\bar{g}, g\bar{r}, g\bar{b}, b\bar{g}, b\bar{r} \\ r\bar{r}, g\bar{g}, b\bar{b}$$

However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$ is **colourless** and does not take part in the strong interaction.

Colour at Work

EXAMPLE: $q\bar{q}$ Annihilation



Normally do not show colour on Feynman diagrams - colour is conserved.

QED POTENTIAL:

$$V_{\text{QED}} = -\frac{\alpha}{r}$$

QCD POTENTIAL:

At short distances QCD potential looks similar

$$V_{\text{QCD}} = -\frac{4}{3} \frac{\alpha_s}{r}$$

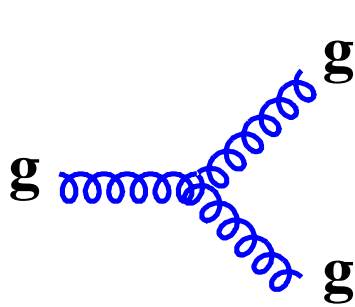
apart from $\frac{4}{3}$ colour factor.

Note: the colour factor ($4/3$) arises because more than one gluon can participate in the process $q \rightarrow qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states

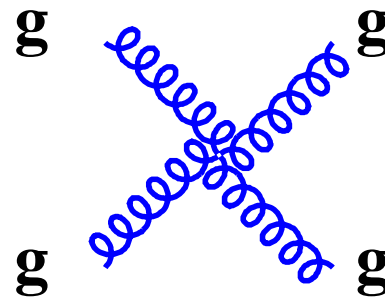
SELF-INTERACTIONS

At this point, QCD looks like a stronger version of QED. This is true up to a point. However, in practice QCD behaves very differently to QED. The similarities arise from the fact that both involve the exchange of **MASSLESS** spin-1 bosons. The big difference is that **GLUONS** carry **colour** “charge”.

GLUONS CAN INTERACT WITH OTHER GLUONS:

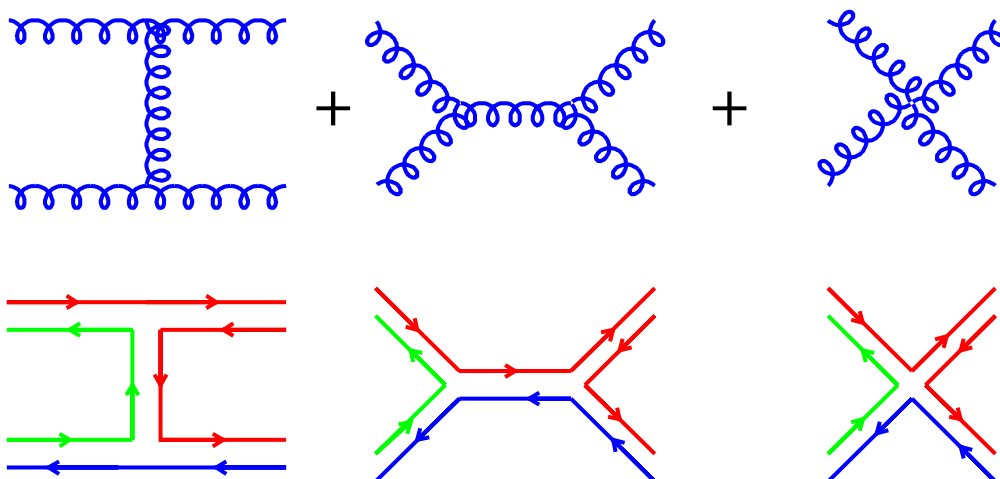


3 GLUON VERTEX



4 GLUON VERTEX

EXAMPLE: Gluon-Gluon Scattering $gg \rightarrow gg$



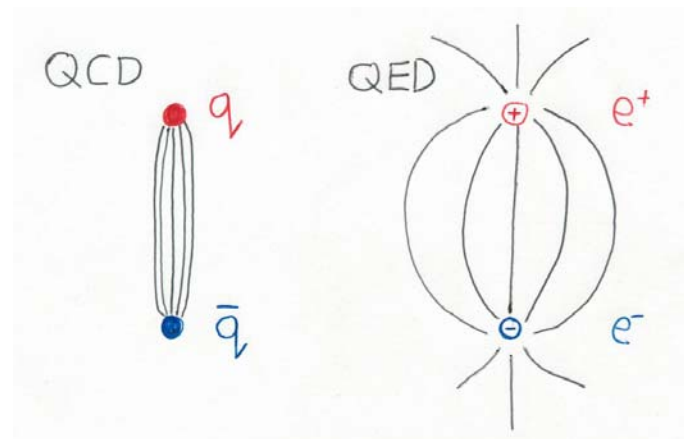
e.g. $r \bar{g} + g \bar{b} \rightarrow r \bar{r} + r \bar{b}$

CONFINEMENT

NEVER OBSERVE: single FREE quarks/gluons

- ★ quarks are always **confined** within hadrons
- ★ This is a consequence of the strong self-interactions of gluons.

Qualitatively, picture the colour field between two quarks. The gluons mediating the force act as additional sources of the colour field - they attract each other. The gluon-gluon interaction pulls the lines of colour force into a narrow tube or **STRING**. In this model the string has a 'tension' and as the quarks separate the string stores potential energy.



Energy stored per unit length \sim constant.

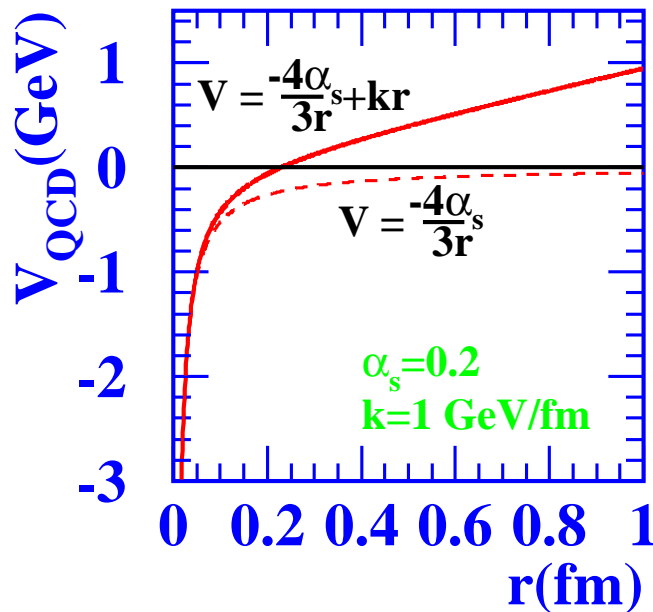
$$V(r) \propto r$$

- ★ Requires infinite energy to separate two quarks. Quarks always come in combinations with zero net colour charge: **CONFINEMENT**.

How Strong is Strong ?

QCD Potential between quarks has two components:

- ★ “COULOMB”-LIKE TERM : $-\frac{4}{3} \frac{\alpha_S}{r}$
- ★ LINEAR TERM : $+kr$



Force between two quarks at separated by 10 m:

$$V_{QCD} = -\frac{\alpha_S}{r} + kr$$

with $k \approx 1 \text{ GeV/fm}$

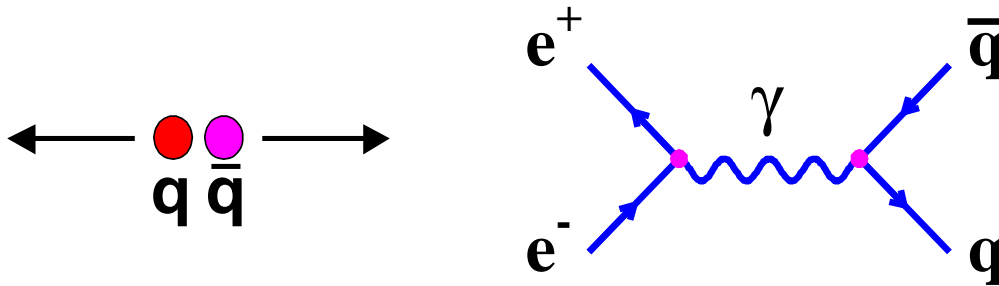
$$F = -\frac{dV}{dr} = \frac{\alpha_S}{r^2} + k$$

$$\begin{aligned} \text{at large } r \quad F &= k = \frac{1.6 \times 10^{-10}}{10^{-15}} \text{ N} \\ &= 160000 \text{ N} \end{aligned}$$

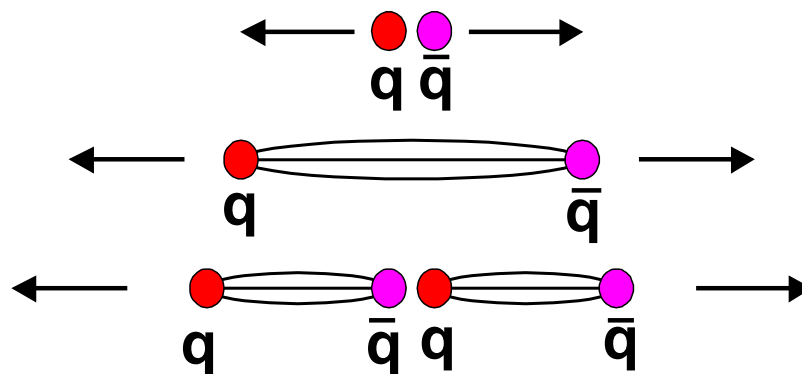
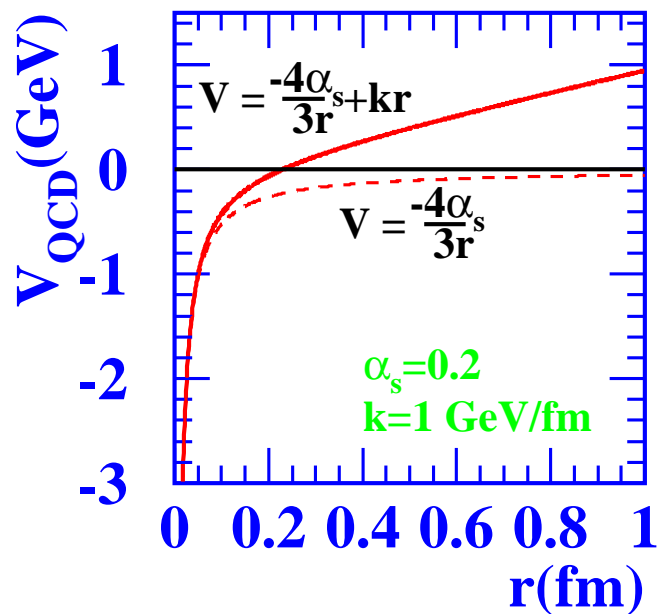
Equivalent to the weight of approximately 65 Widdecombes.

JETS

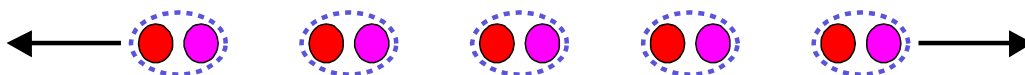
Consider the $q\bar{q}$ pair produced in $e^+e^- \rightarrow q\bar{q}$:



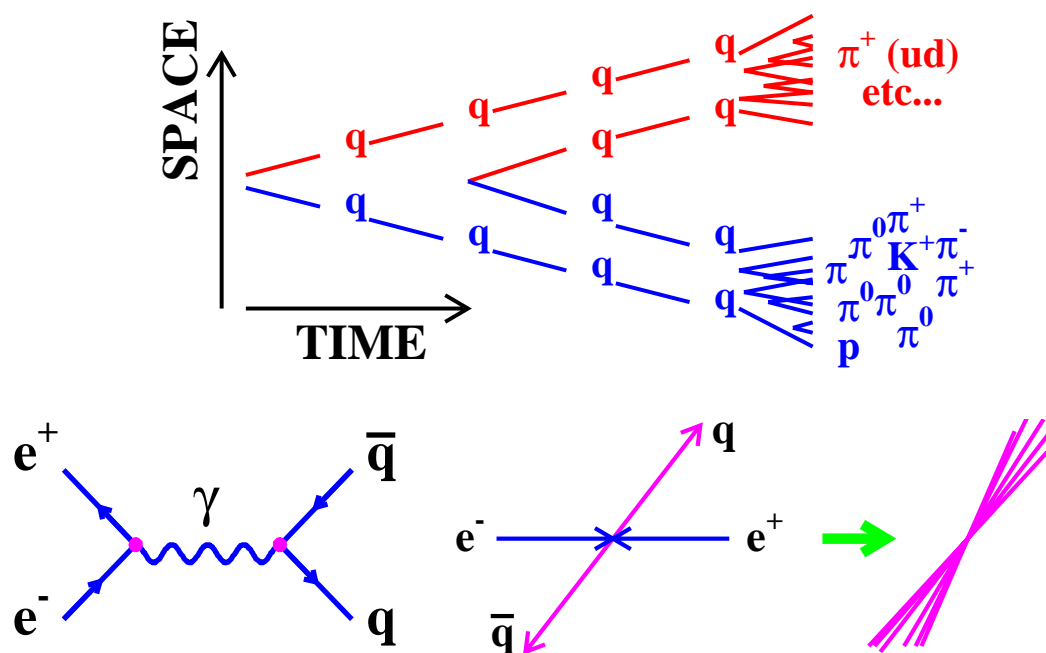
As the quarks separate, the energy stored in the colour field ('string') starts to increase linearly with separation. When $E_{stored} > 2m_q$ new $q\bar{q}$ pairs can be created.



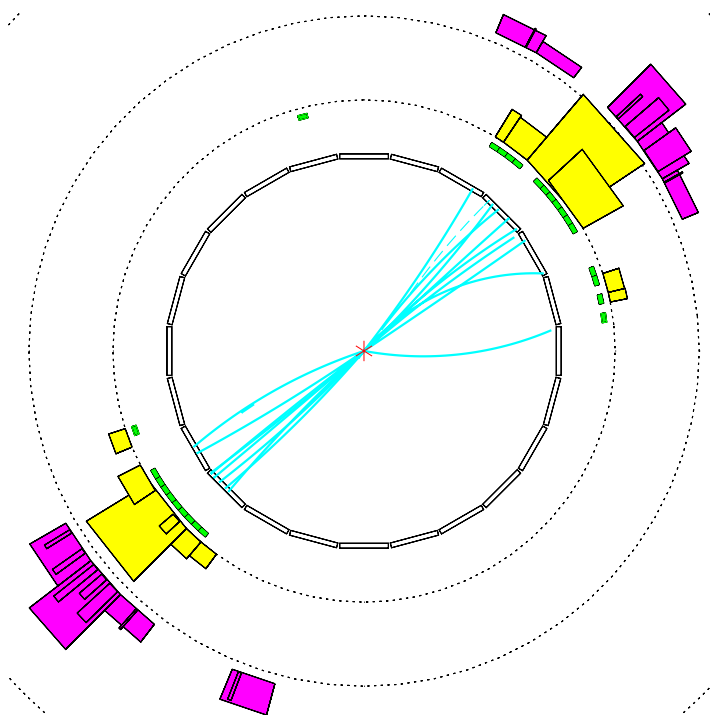
as energy decreases... hadrons freeze out



As quarks separate, more $q\bar{q}$ pairs are produced from the potential energy of the colour field. This process is called **HADRONIZATION**. Start out with **quarks** and end up with narrowly collimated **JETS** of **HADRONS**

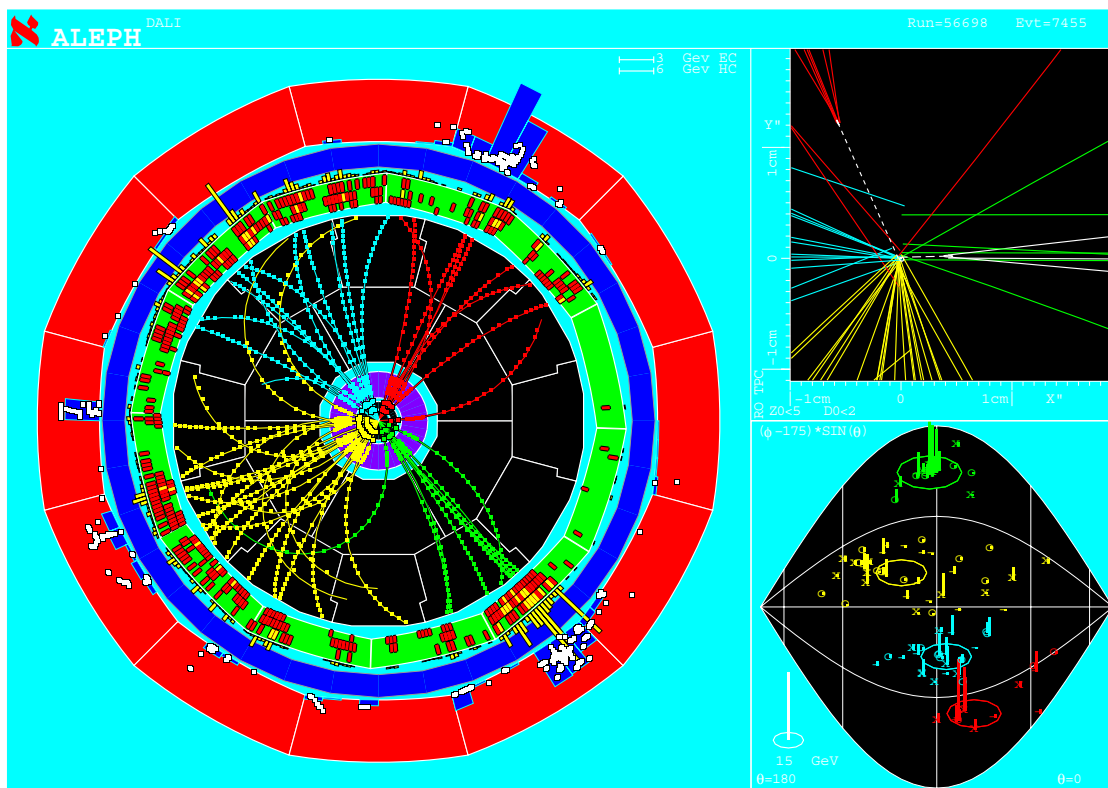


Typical $e^+e^- \rightarrow q\bar{q}$ Event



The hadrons in a quark(anti-quark) jet follow the direction of the original quark(anti-quark). Consequently $e^+e^- \rightarrow q\bar{q}$ is observed as a pair of **back-to-back** jets of hadrons

aside.....



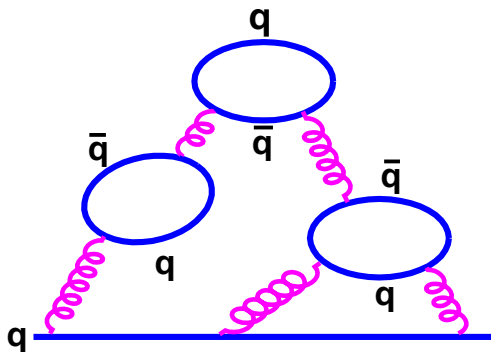
★ You will now recognize the “Higgs” event from the cover of Handout I as

$$e^+e^- \rightarrow \text{something} \rightarrow q\bar{q}q\bar{q}$$

Running of α_S

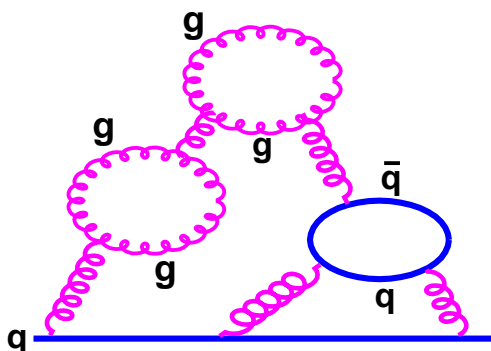
- ★ α_S specifies the strength of the strong interaction
- ★ BUT just as in QED, α_S isn't a constant, it "runs"
- ★ In QED the bare electron charge is **screened** by a cloud of virtual electron-positron pairs.
- ★ In QCD a similar effect occurs.

In QCD quantum fluctuations lead to a 'cloud' of virtual $q\bar{q}$ pairs



one of many (an infinite set) such diagrams analogous to those for QED.

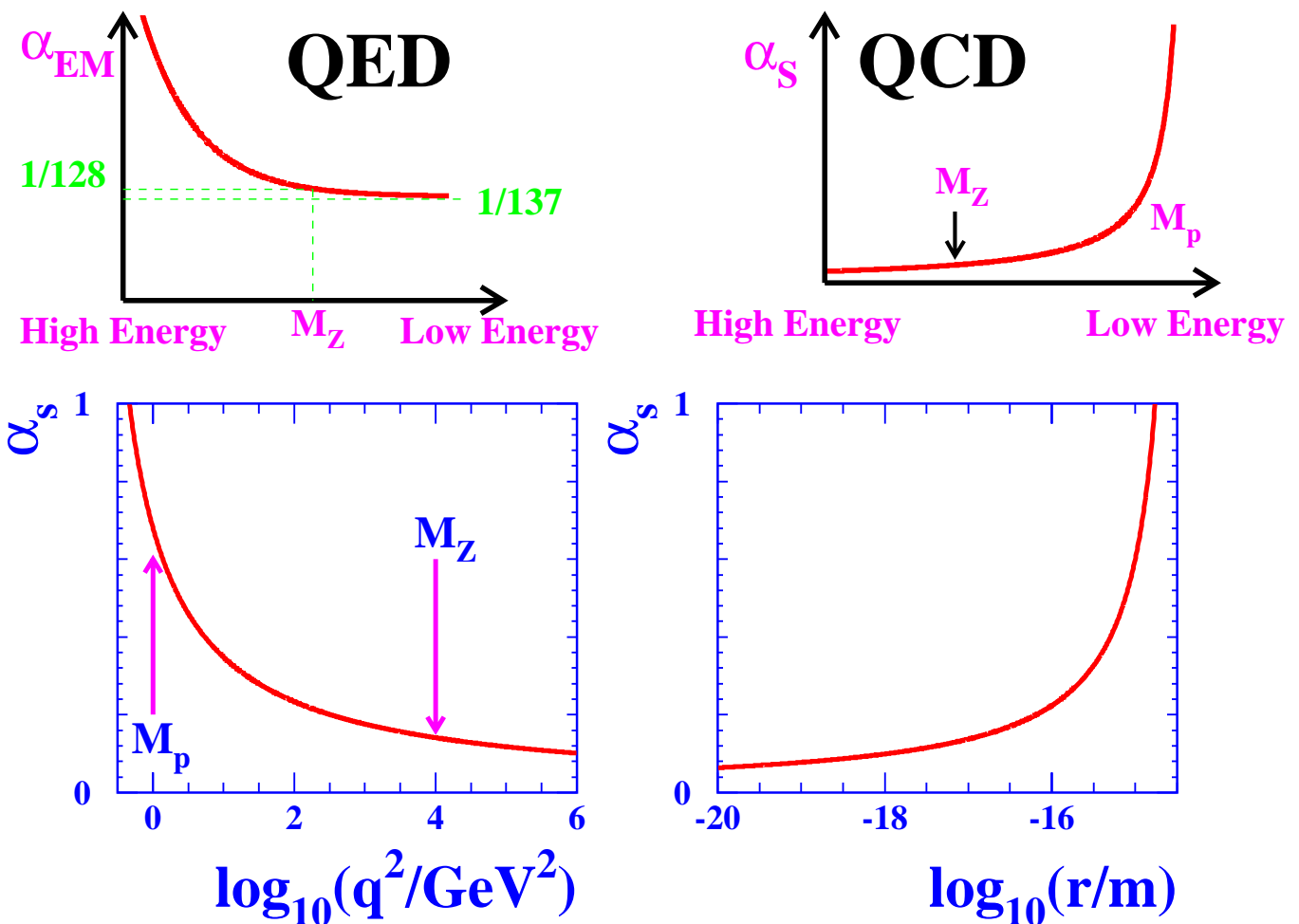
In QCD the gluon **self-interactions** ALSO lead to a 'cloud' of virtual gluons



one of many (an infinite set) such diagrams. Here there is no analogy in QED, photons don't have self-interactions since they don't carry the charge of the interaction.

Colour Anti-Screening

- ★ Due to the gluon self-interactions bare colour charge is screened by both virtual quarks and virtual gluons
- ★ The cloud of virtual gluons carries colour charge and the effective colour charge **INCREASES** with distance !
- ★ At low energies (large distances) α_S becomes large \rightarrow can't use perturbation theory (**not a weak perturbation**)



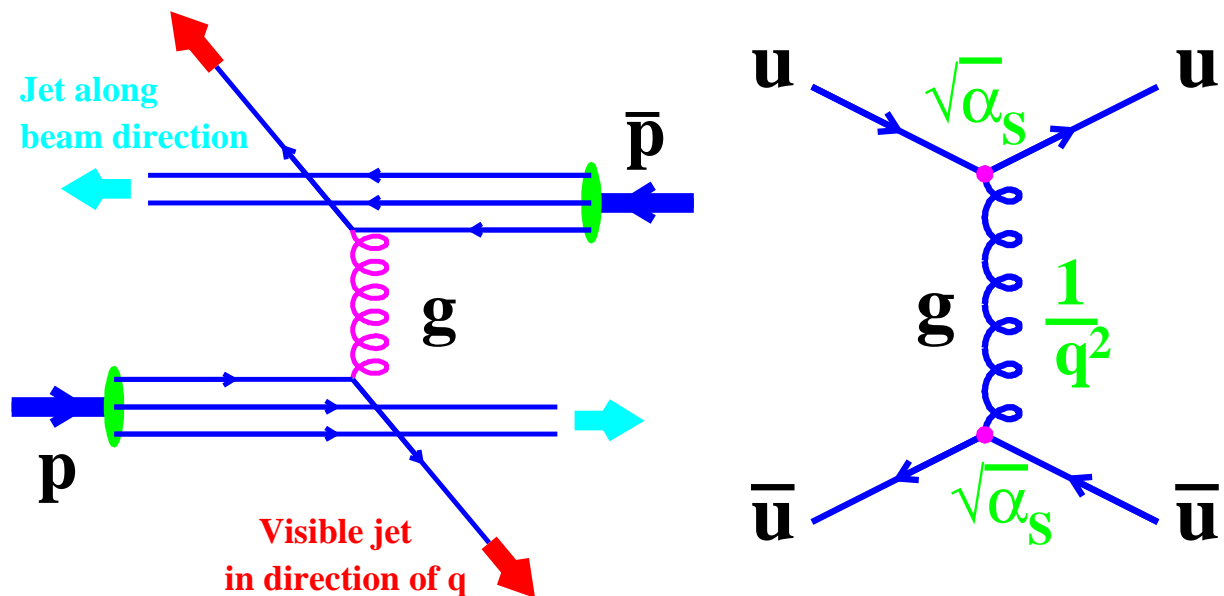
- ★ At High energies (short distances) α_S is small. In this regime treat quarks as free particles and can use perturbation theory

ASYMPTOTIC FREEDOM

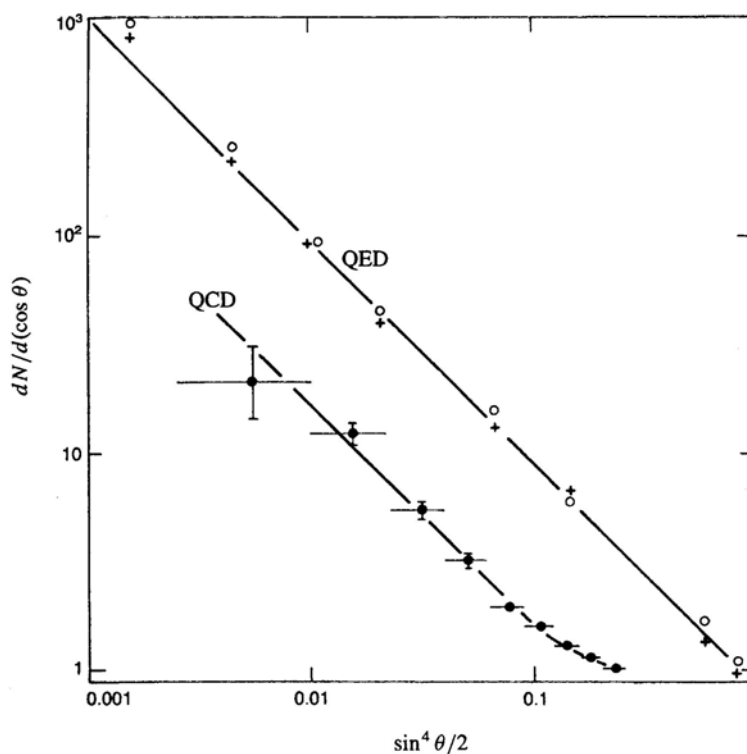
- ★ At $\sqrt{s} = 100 \text{ GeV}$, $\alpha_S = 0.12$.

Scattering in QCD

EXAMPLE: High energy proton-antiproton scattering

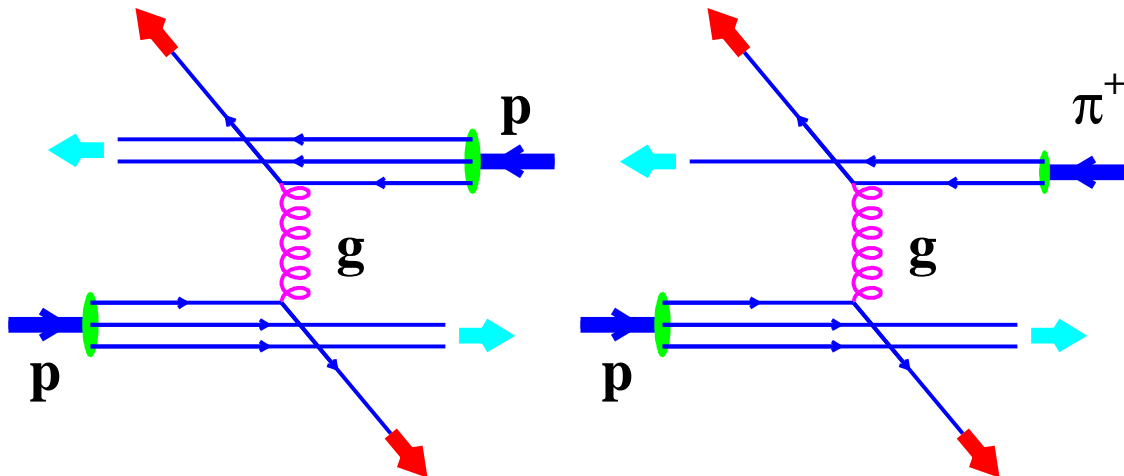


$$M \sim \frac{1}{q^2} \sqrt{\alpha_s} \sqrt{\alpha_s} \Rightarrow \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_s)^2}{\sin^4 \theta/2}$$



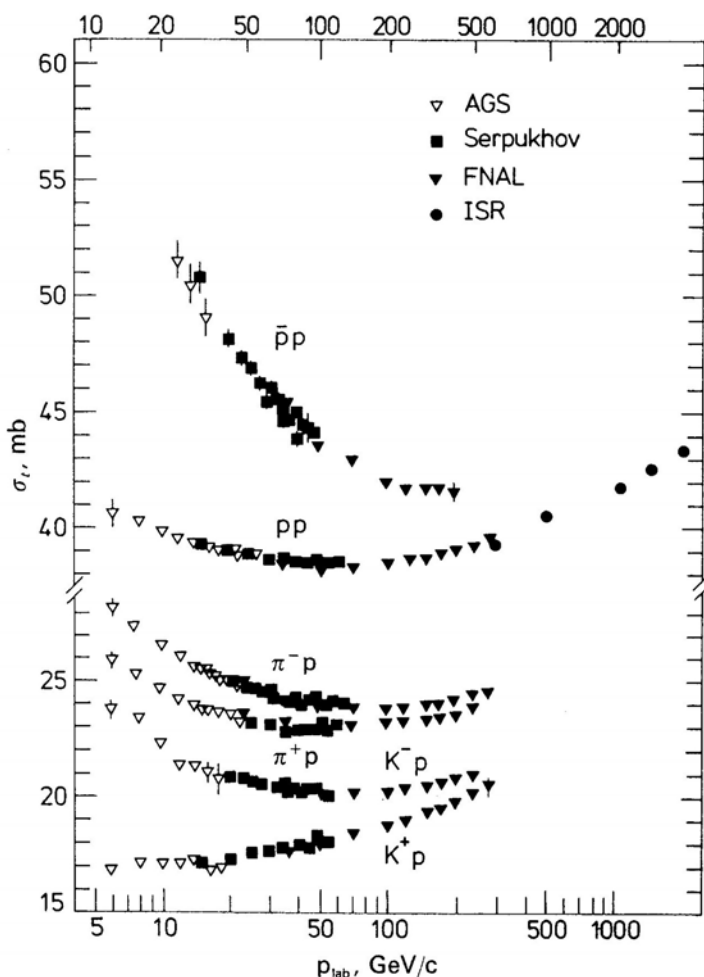
The upper points are the Geiger and Marsden data (1911) for the elastic scattering of α particles as they traverse thin gold and silver foils. The lower points show the angular distribution of the quark jets observed in proton-antiproton scattering at $q^2 = 2000 \text{ GeV}^2$. Both follow the Rutherford formula for elastic scattering: $\sin^4 \frac{\theta}{2}$.

EXAMPLE: pp vs $\pi^+ p$ scattering



Calculate ratio of $\sigma(pp)_{\text{total}}$ to $\sigma(\pi^+ p)_{\text{total}}$

★ QCD does not distinguish between quark flavours, only COLOUR charge of quarks matters.



At high energy ($E \gg$ binding energy of quarks within hadrons) ratio of pp and $p\pi$ total cross sections depends on number of possible quark-quark combinations.

Predict

$$\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2 \times 3}{3 \times 3} = \frac{2}{3}$$

Experiment

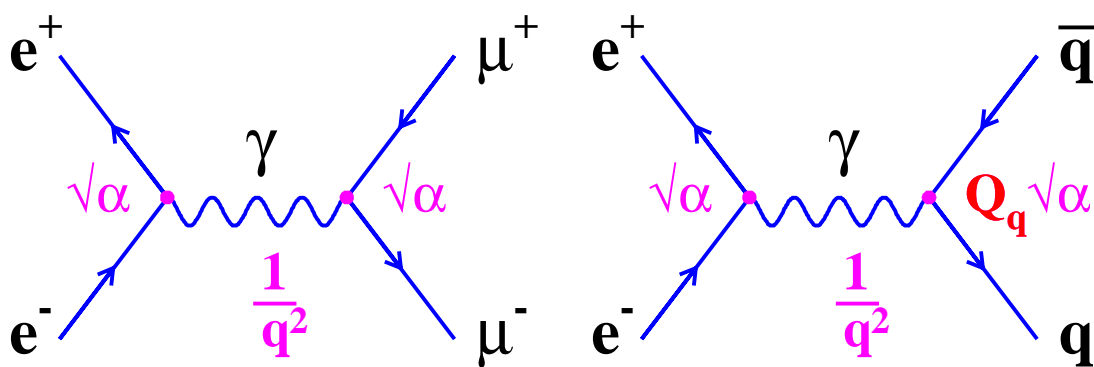
$$\frac{\sigma(\pi p)}{\sigma(pp)} \approx \frac{24 \text{ mb}}{38 \text{ mb}} \approx 0.63$$

QCD in e^+e^- Annihilation

Direct evidence for the existence of colour comes from e^+e^- Annihilation.

★ Compare $e^+e^- \rightarrow \mu^+\mu^-$, $e^+e^- \rightarrow q\bar{q}$:

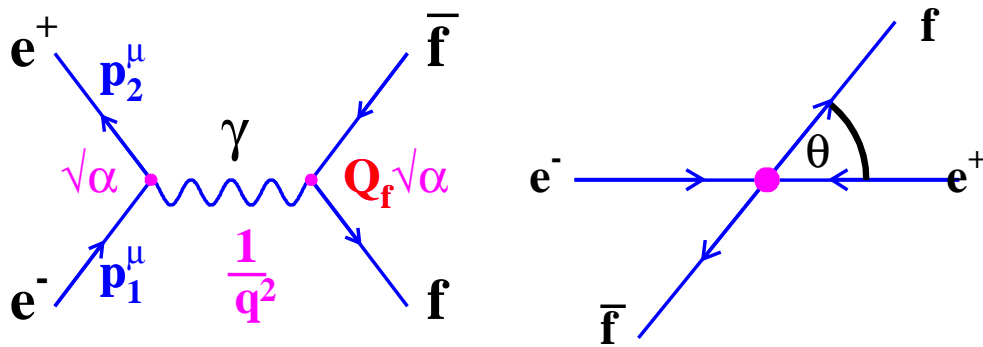
$$R_\mu = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



If we neglect the masses of the **final state** quarks/muons then the **ONLY** difference is the **charge** of the final state particles ($Q_\mu = -1$, $Q_q = +\frac{2}{3}$ or $-\frac{1}{3}$)

Start by calculating the cross section for the process $\sigma(e^+e^- \rightarrow f\bar{f})$. ($f\bar{f}$ represent a fermion-antifermion pair e.g. $\mu^+\mu^-$ or $q\bar{q}$).

see Handout II for the case where $f\bar{f} = \mu^+\mu^-$



Electron/Positron beams along z -axis

$$p_1^\mu = (E, p_x, p_y, p_z)$$

$$p_1^\mu = (E, 0, 0, E) \text{ neglecting } m_e$$

$$p_2^\mu = (E, 0, 0, -E)$$

$$q^\mu = p_1^\mu + p_2^\mu$$

$$= (2E, 0, 0, 0)$$

$$q^2 = 4E^2 = s$$

where s is (centre-of-mass energy)².

Fermi's Golden rule and Born Approximation

$$\frac{d\sigma}{d\Omega} = 2\pi |M|^2 \frac{d\rho(E_f)}{d\Omega}$$

Matrix element M :

$$M = \langle v_{e^+} | Q_e | u_{e^-} \rangle \frac{1}{q^2} \langle v_{\bar{f}} | Q_f | u_f \rangle$$

$$= \frac{-4\pi\alpha Q_e Q_f}{q^2} \quad \text{with} \quad \alpha = \frac{e^2}{4\pi}$$

$$\frac{d\sigma}{d\Omega} = 2\pi \frac{(-4\pi\alpha Q_e Q_f)^2}{q^4} \frac{E^2}{(2\pi)^2} \frac{1}{4} (1 + \cos^2 \theta)$$

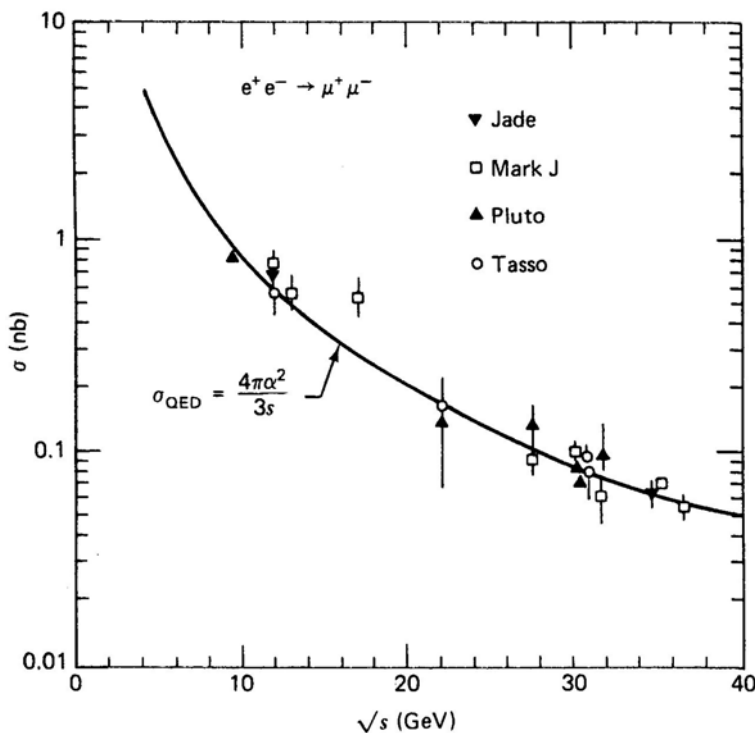
$$= \frac{\alpha^2 Q_f^2}{4s} (1 + \cos^2 \theta)$$

★ $(1 + \cos^2 \theta)$ comes from spin-1 photon “decaying” to two spin-half fermions. see lecture on Dirac equation

Total cross section for $e^+e^- \rightarrow f\bar{f}$

$$\begin{aligned}
 \sigma &= \int \frac{d\sigma}{d\Omega} d\Omega \\
 &= \int_0^{2\pi} \int_0^\pi \frac{\alpha^2 Q_f^2}{4s} (1 + \cos^2 \theta) \sin \theta d\theta d\phi \\
 &= \frac{\pi \alpha^2 Q_f^2}{2s} \int_{-1}^{+1} (1 + y^2) dy \quad (y = \cos \theta) \\
 &= \frac{4\pi \alpha^2 Q_f^2}{3s}
 \end{aligned}$$

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$



$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$
for e^+e^- collider data
at centre-of-mass ener-
gies 8-36 GeV

Back to

$$R_\mu = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

For a **single** quark flavour of a **given colour**

$$R = Q_q^2$$

However, we measure $e^+e^- \rightarrow \text{jets}$ not $e^+e^- \rightarrow u\bar{u}$. A jet from a **u**-quark looks just like a jet from a **d**-quark... Need to sum over **flavours** (u,d,c,s,t,b) and colours (**r**, **g**, **b**).

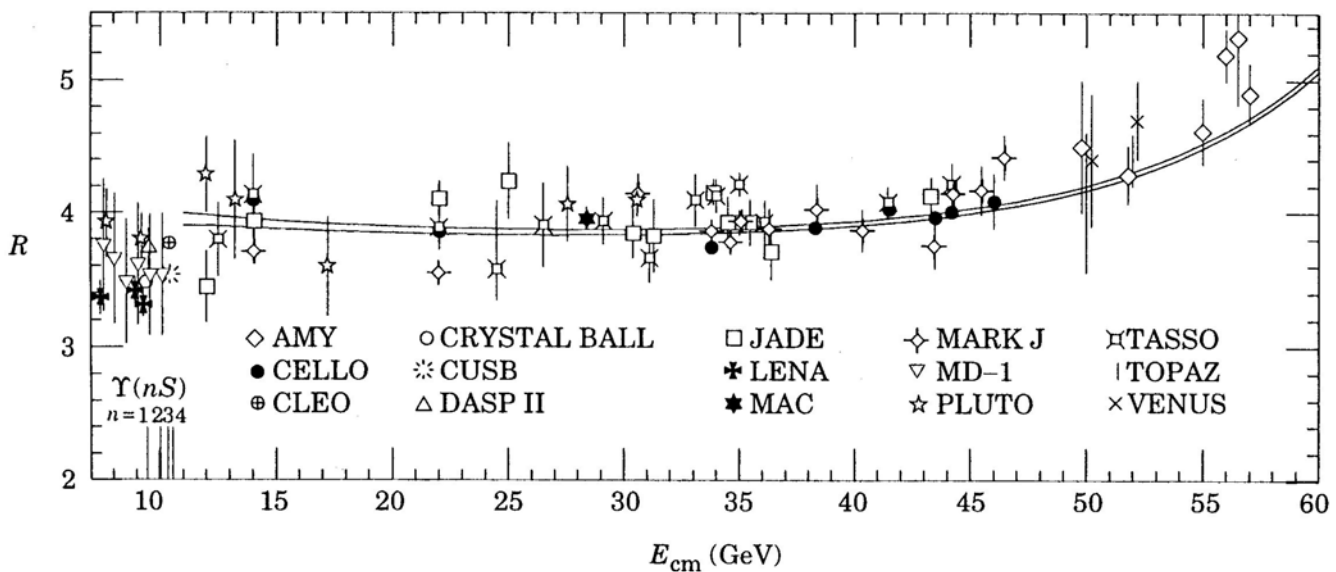
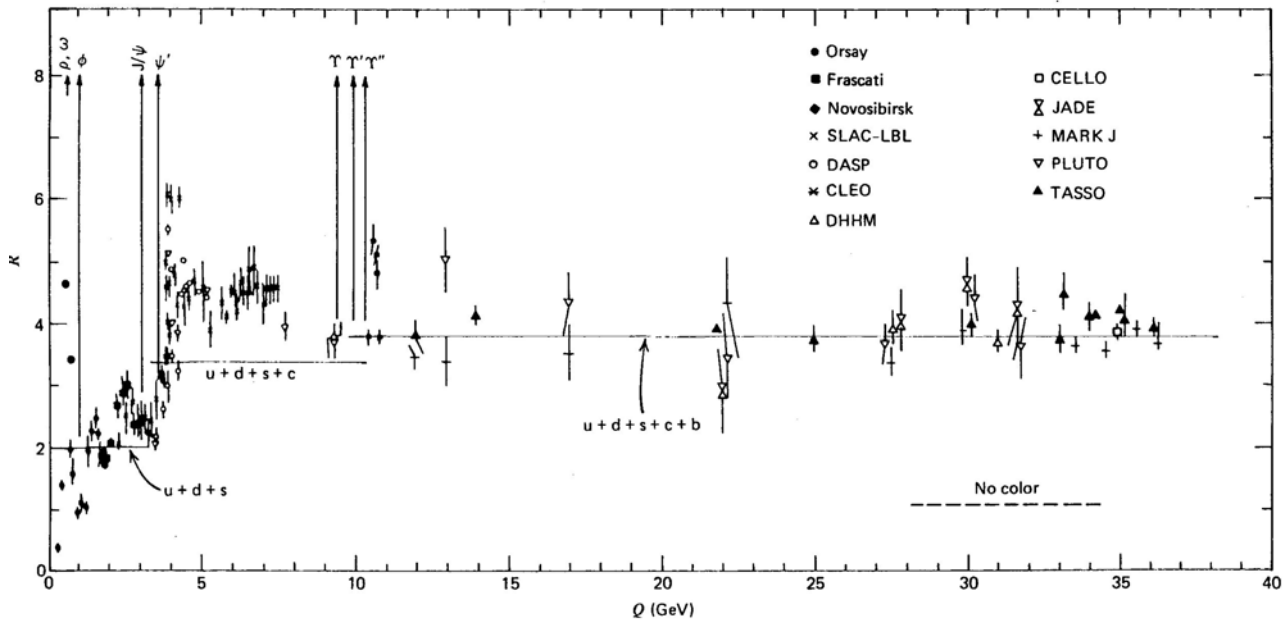
$$R = 3 \sum_i Q_i^2 \quad (3\text{colours})$$

where the sum is over all quark flavours **kinematically** accessible at centre-of-mass energy, \sqrt{s} , of collider.

Energy	Ratio R
$\sqrt{s} > 2m_s \sim 1 \text{ GeV}$	$3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$ u,d,s
$\sqrt{s} > 2m_c \sim 4 \text{ GeV}$	$3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = 3\frac{1}{3}$ u,d,s,c
$\sqrt{s} > 2m_b \sim 10 \text{ GeV}$	$3\left(\dots + \frac{1}{9}\right) = 3\frac{2}{3}$ u,d,s,c,b
$\sqrt{s} > 2m_t \sim 350 \text{ GeV}$	$3\left(\dots + \frac{4}{9}\right) = 5$ u,d,s,c,b,t

$$R_\mu = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

Data: \sqrt{s} from 0 – 40 GeV



★ R_μ increases in steps with \sqrt{s}

★ $\sqrt{s} < 11 \text{ GeV}$ region complicated by resonances: charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$).

★ R_μ Data exclude 'no colour' hypothesis.

STRONG EVIDENCE for COLOUR

Experimental Evidence for Colour

★ R_μ

★ The existence of the $\Omega^- (sss)$

The $\Omega^- (sss)$ is a ($L=0$) spin- $\frac{3}{2}$ baryon consisting of 3 strange-quarks. The wave-function

$$\psi = s \uparrow s \uparrow s \uparrow$$

is SYMMETRIC under particle interchange.

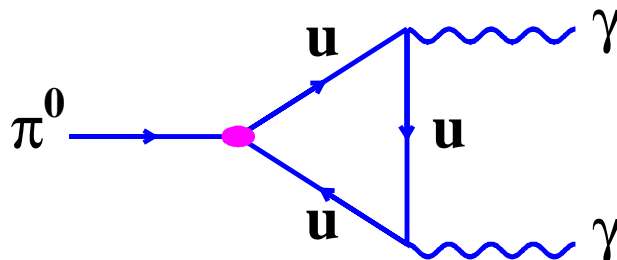
However quarks are FERMIONS, therefore require an ANTI-SYMMETRIC wave-function, *i.e.* need another degree of freedom, namely COLOUR.

$$\psi = (s \uparrow s \uparrow s \uparrow) \psi_{colour}$$

$$\psi_{colour} = \frac{1}{\sqrt{6}} (rgb + gbr + brg - grb - rbg - bgr)$$

★ $\pi^0 \rightarrow \gamma\gamma$ decay rate

Need colour to explain $\pi^0 \rightarrow \gamma\gamma$ decay rate.



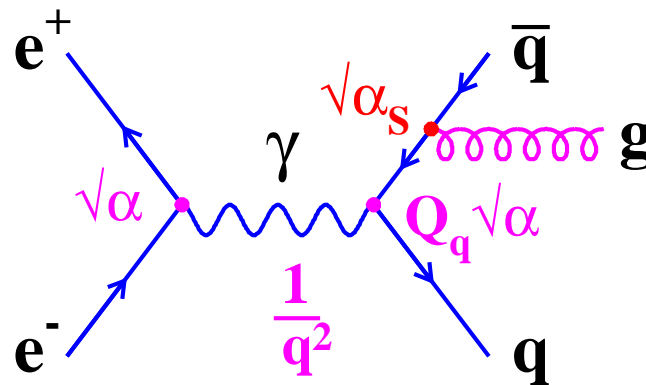
$$\Gamma(\pi^0 \rightarrow \gamma\gamma) \propto N_{colour}^2$$

$$\text{EXPT : } N_{colour} = 2.99 \pm 0.12$$

Evidence for Gluons

In QED, electrons can radiate photons. In QCD quarks can radiate gluons.

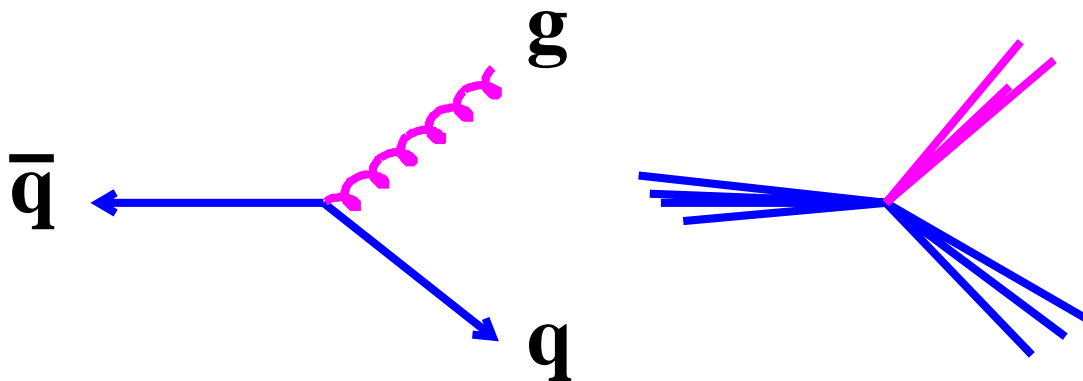
$$\underline{e^+ e^- \rightarrow q \bar{q} g}$$



giving an extra factor of $\sqrt{\alpha_s}$ in the matrix element, i.e. an extra factor of α_s in cross section.

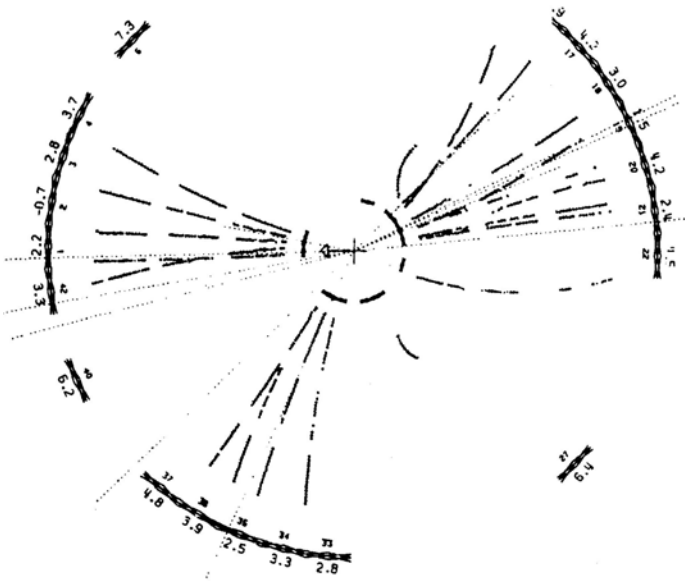
In QED we can detect the photons. In QCD we never see free gluons due to confinement.

Experimentally detect gluons as an additional jet:
3-Jet Events.

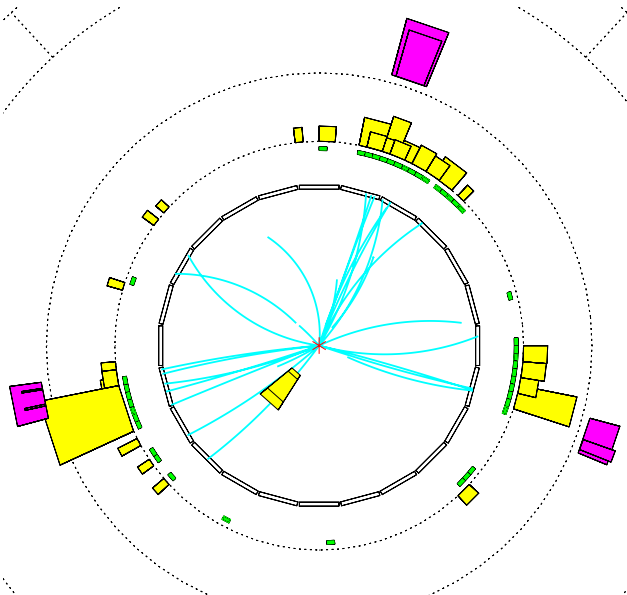


★ Angular distribution of gluon jet depends on gluon spin

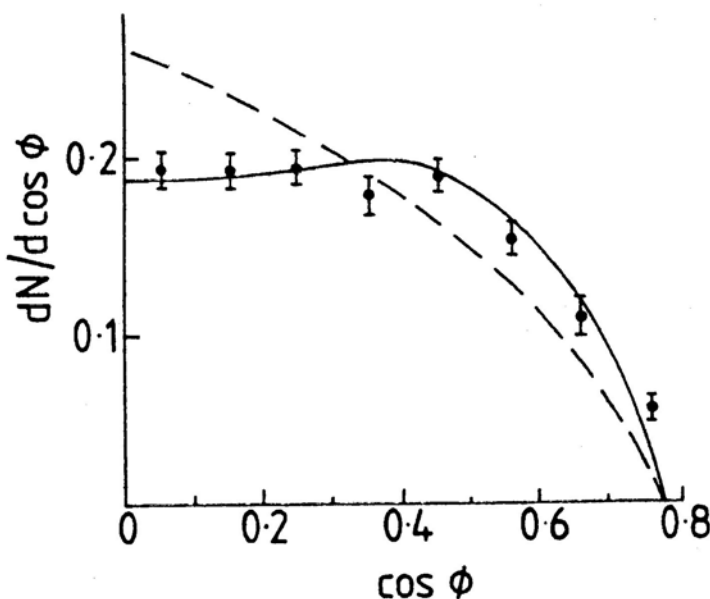
3-Jet Events and Gluon Spin



JADE $\sqrt{s} = 31$ GeV
Direct Evidence for Gluons (1978)



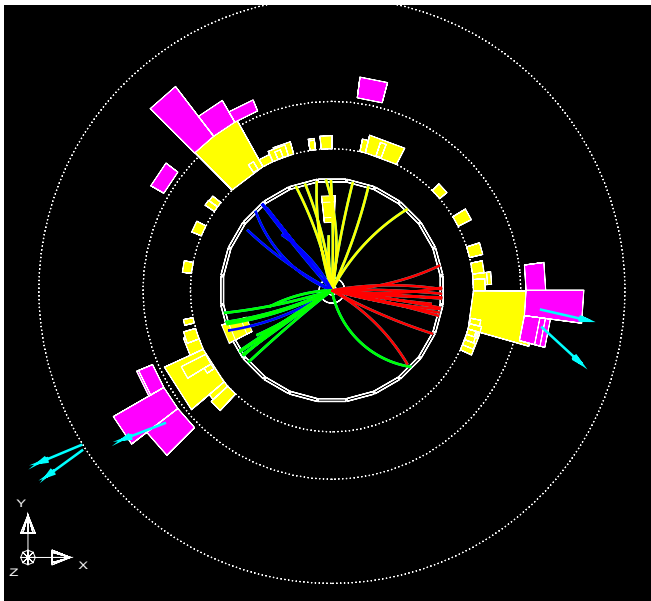
OPAL $\sqrt{s} = 91$ GeV
(1990)



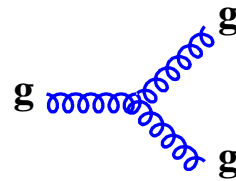
Distribution of the angle, ϕ between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cms frame). ϕ depends on the spin of the gluon.

⇒ **GLUON is SPIN-1**

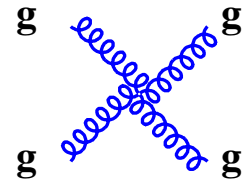
Gluon Self-Interactions



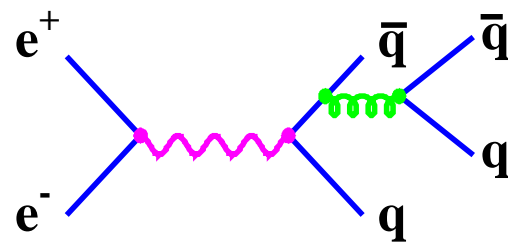
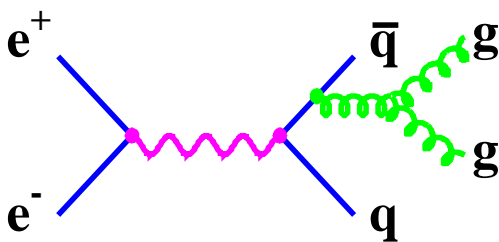
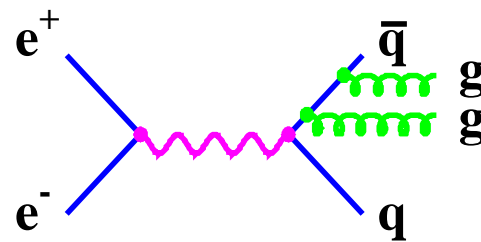
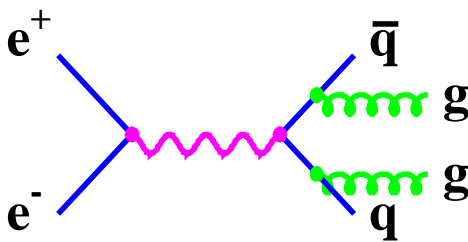
Direct Evidence for the existence of the gluon self-interactions from 4-JET events.



3 GLUON VERTEX



4 GLUON VERTEX

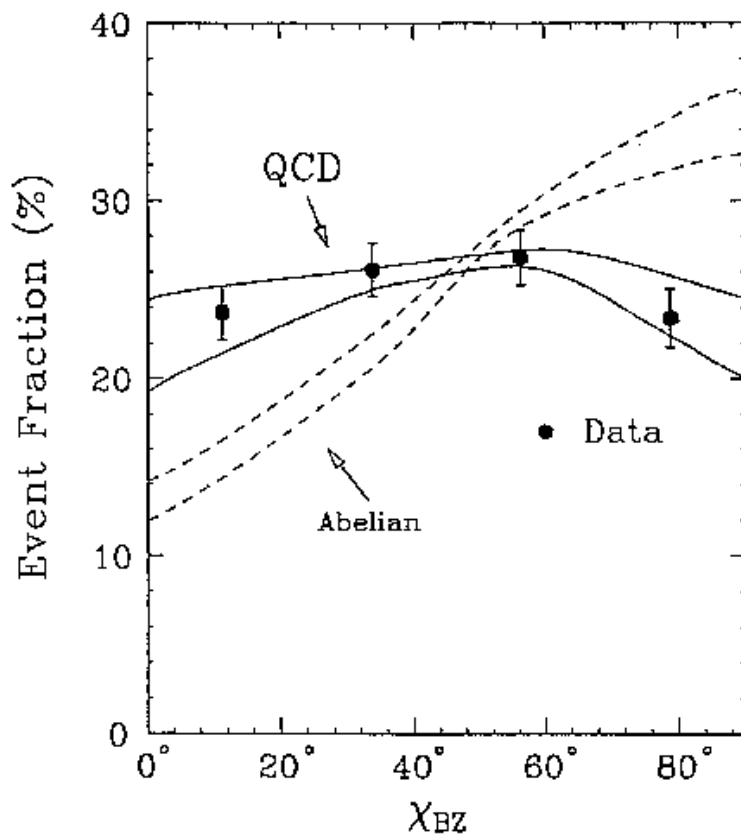
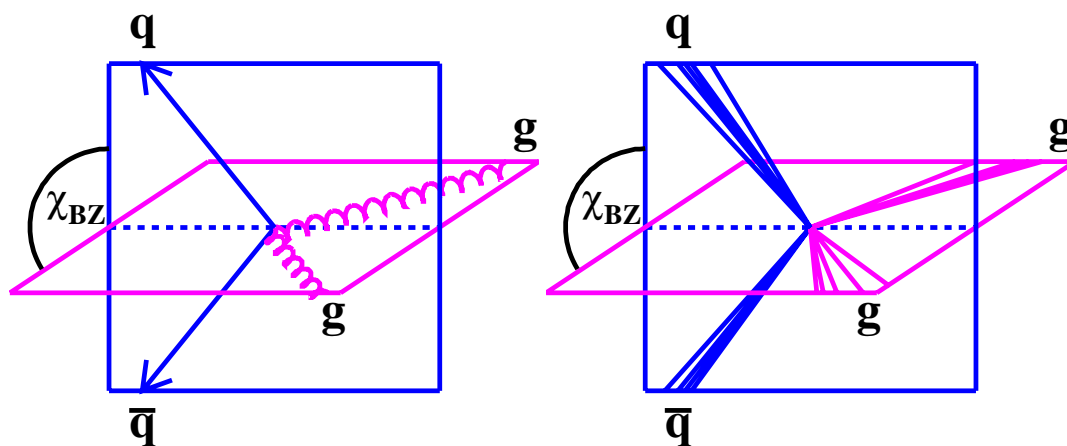


★ Angular distribution of jets is sensitive to existence triple gluon vertex:

- ★ $q\bar{q}g$ vertex consists of 2 spin- $\frac{1}{2}$ quarks and a spin-1 gluon.
- ★ ggg vertex consists of 3 spin-1 gluons, \therefore different angular distribution.

Experimentally:

- ★ Define the two lowest energy jets as the gluons. (gluon jets are more likely to be low energy than quark jets)
- ★ Measure angle between the plane containing the 'quark' jets and the plane containing the 'gluon' jets, χ_{BZ}

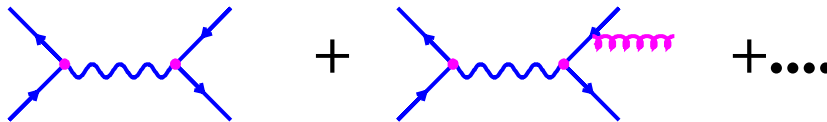


Gluon self-interactions are required to describe the experimental data. Theory without self-interactions (ABELIAN) is inconsistent with observations

Measuring α_S

α_S can be measured in many ways. The cleanest is from

R_μ : In practice measure



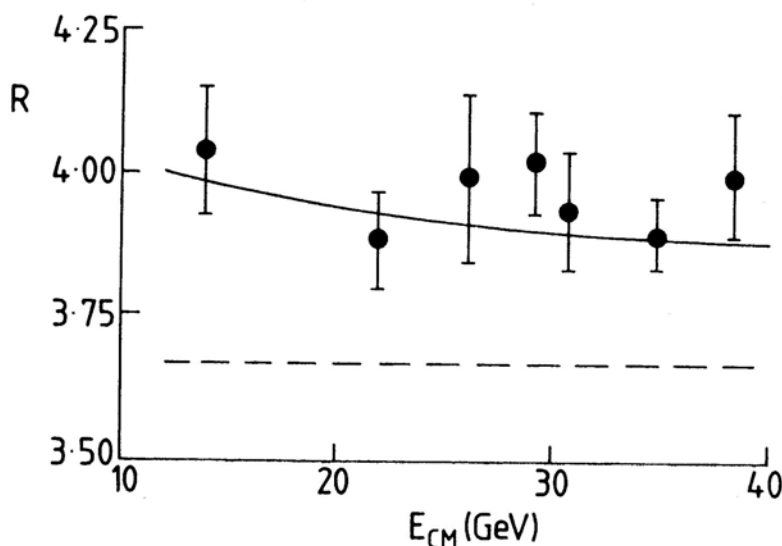
i.e. don't distinguish 2/3 jets. So measure

$$R_\mu = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

not $R_\mu = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$

When gluon radiation is included :

$$R_\mu = 3 \sum Q_q^2 \left(1 + \frac{\alpha_S}{\pi} \right)$$



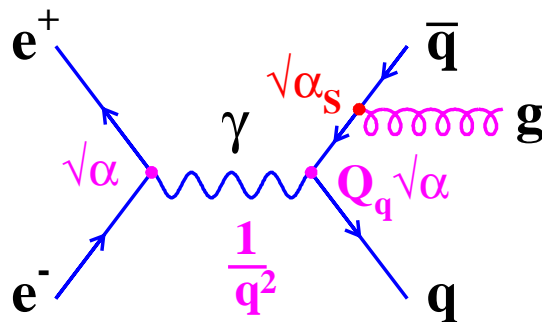
$$\sum_q Q_q^2 = 3\frac{2}{3}$$

Therefore $\left(1 + \frac{\alpha_S}{\pi} \right) \approx \frac{3.9}{3.66}$

giving $\alpha_S(q^2 = 25^2) \approx 0.20$

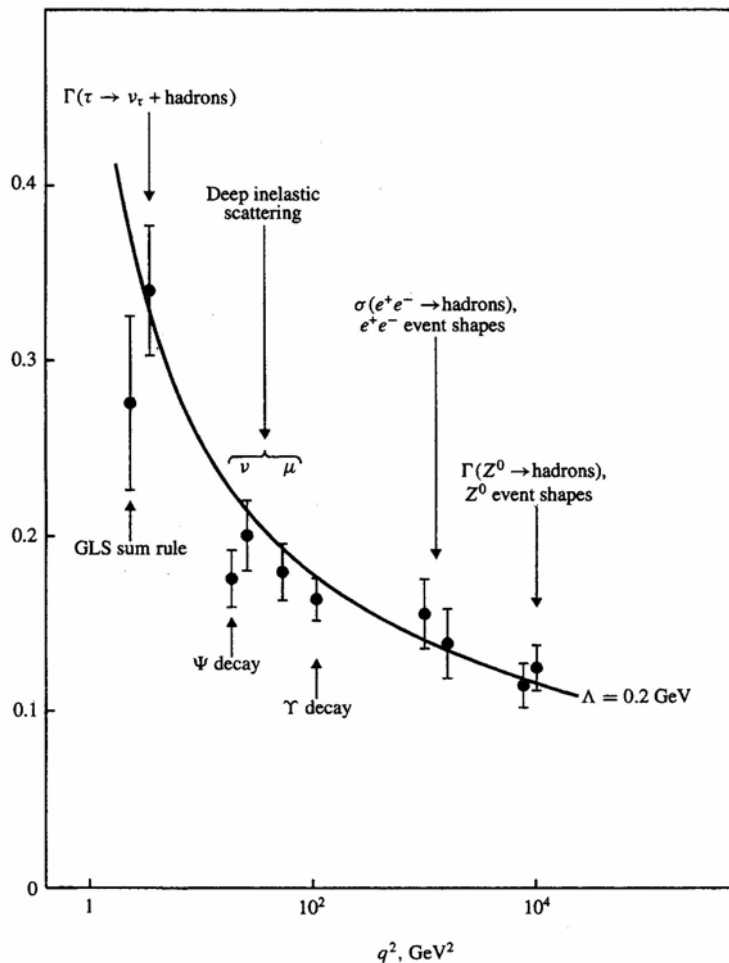
Many other ways to measure α_S

e.g. 3 jet rate : $e^+e^- \rightarrow q\bar{q}g$



$$\frac{\sigma(3 \text{ jet})}{\sigma(2 \text{ jet})} = \frac{\sigma(q\bar{q}g)}{\sigma(q\bar{q})} \propto \alpha_S$$

α_S Summary



Summary of α_S measurements

α_S RUNS !

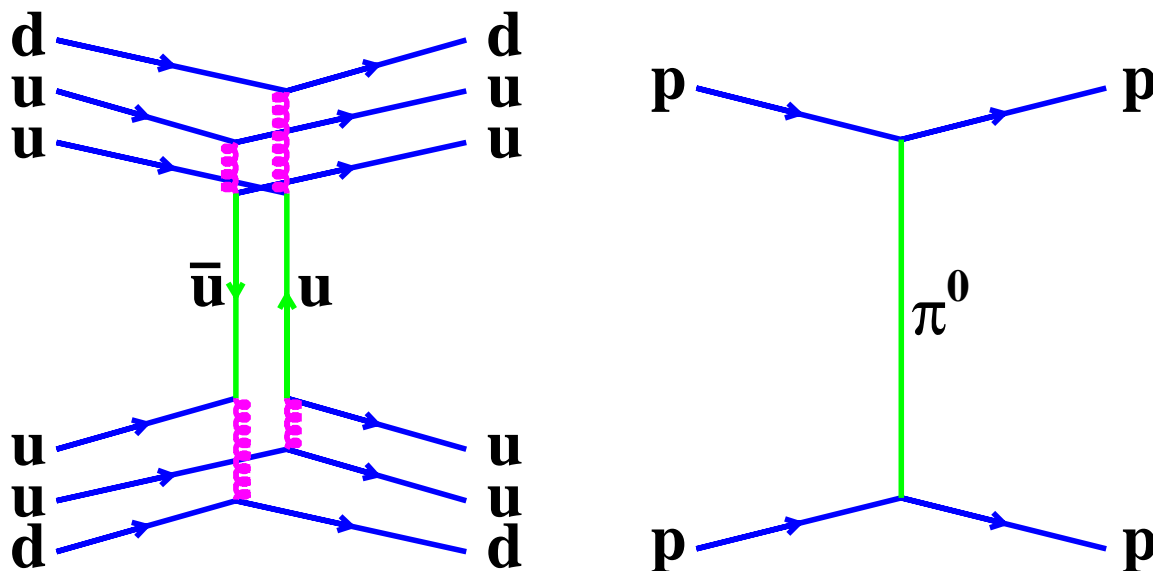
Nucleon-Nucleon Interactions

★ Bound qqq states (e.g. Protons and Neutrons) are COLOURLESS (COLOUR SINGLETs).

★ They can only interact via COLOURLESS intermediate states - i.e. not by single gluons. (conservation of colour charge)

★ Interact by exchange of PIONS

★ One possible diagram shown below :



★ Nuclear potential is YUKAWA potential with

$$V(\tilde{r}) = -\frac{g^2}{4\pi r} e^{-m_\pi r}$$

★ Short range force : range $\sim (m_\pi)^{-1}$

$$\begin{aligned} \text{Range } R &= (0.140 \text{ GeV})^{-1} \\ &= 7 \text{ GeV}^{-1} \\ &= 7\hbar c / (1.6 \times 10^{-10}) \text{ m} \\ &= 1.4 \text{ fm} \end{aligned}$$