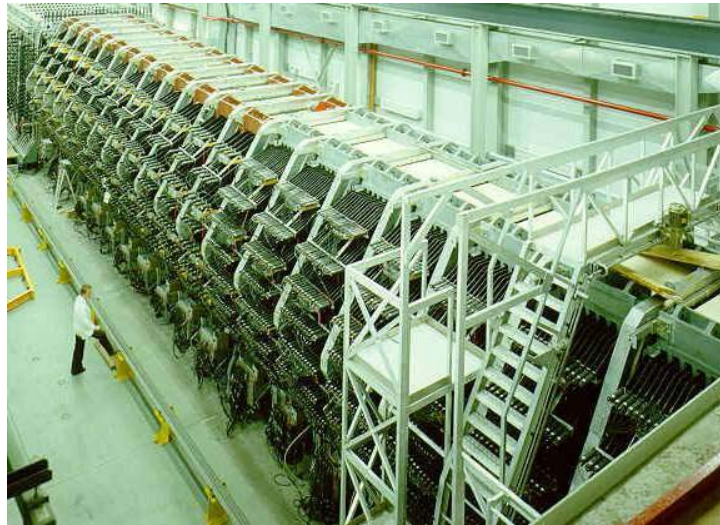


Particle Physics

Michaelmas Term 2009
Prof Mark Thomson



Handout 10 : Leptonic Weak Interactions and Neutrino Deep Inelastic Scattering

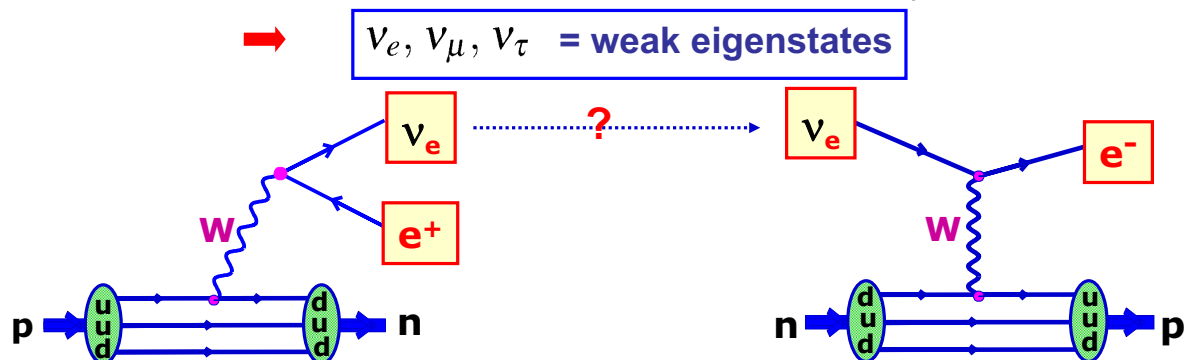
Prof. M.A. Thomson

Michaelmas 2009

305

Aside : Neutrino Flavours

- ★ Recent experiments (see Handout 11) \rightarrow neutrinos have mass (albeit very small)
- ★ The textbook neutrino states, ν_e, ν_μ, ν_τ , are not the fundamental particles; these are ν_1, ν_2, ν_3
- ★ Concepts like “electron number” conservation are now known **not** to hold.
- ★ So what are ν_e, ν_μ, ν_τ ?
- ★ Never **directly** observe neutrinos – can only detect them by their weak interactions. Hence by **definition** ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron



- ★ Unless dealing with **very large** distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the **weak interaction** continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

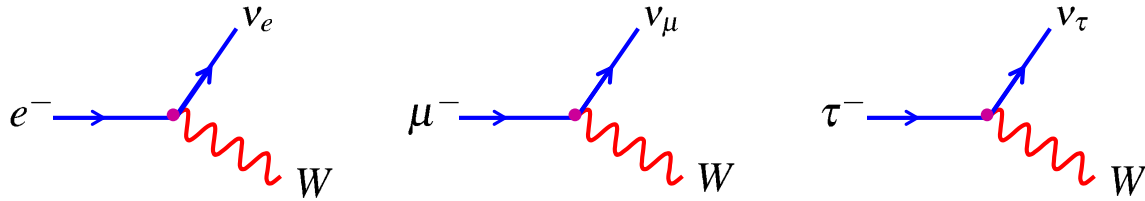
Prof. M.A. Thomson

Michaelmas 2009

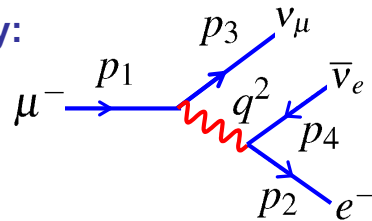
306

Muon Decay and Lepton Universality

★ The leptonic charged current (W^\pm) interaction vertices are:



★ Consider muon decay:



• It is straight-forward to write down the matrix element

$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2} [\bar{u}(p_3) \gamma^\mu (1 - \gamma^5) u(p_1)] g_{\mu\nu} [\bar{u}(p_2) \gamma^\nu (1 - \gamma^5) v(p_4)]$$

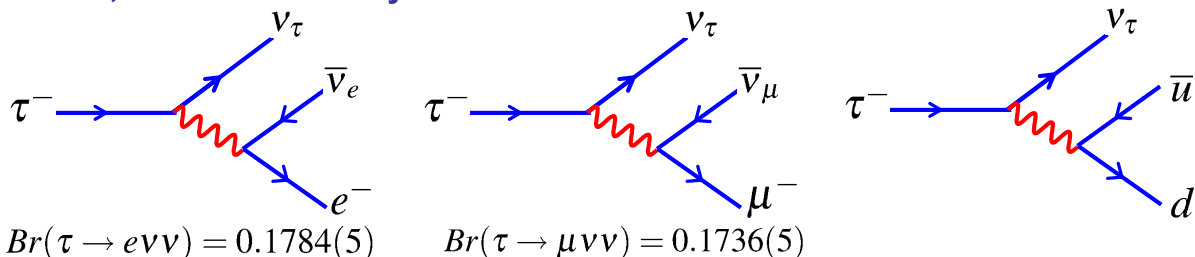
Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$ i.e. in limit of Fermi theory

• Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

• The muon to electron rate $\Gamma(\mu \rightarrow e \nu \nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu}$ with $G_F = \frac{g_W^2}{2\sqrt{2}m_W^2}$

• Similarly for tau to electron $\Gamma(\tau \rightarrow e \nu \nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$

• However, the tau can decay to a number of final states:



$$Br(\tau \rightarrow e \nu \nu) = 0.1784(5)$$

$$Br(\tau \rightarrow \mu \nu \nu) = 0.1736(5)$$

• Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

• Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \rightarrow e \nu \nu) = \Gamma_\tau Br(\tau \rightarrow e \nu \nu) = Br(\tau \rightarrow e \nu \nu) / \tau_\tau$$

• Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \quad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \rightarrow e \nu \nu)$$

- All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94) \text{ GeV} \quad \tau_\mu = 2.19703(4) \times 10^{-6} \text{ s}$$

$$m_\tau = 1.77699(28) \text{ GeV} \quad \tau_\tau = 0.2906(10) \times 10^{-12} \text{ s}$$

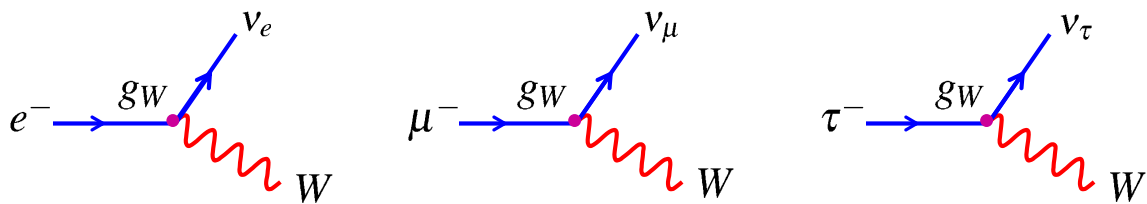
$$Br(\tau \rightarrow e \nu \nu) = 0.1784(5)$$

$$\Rightarrow \frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \rightarrow e \nu \nu) = 1.0024 \pm 0.0033$$

- Similarly by comparing $Br(\tau \rightarrow e \nu \nu)$ and $Br(\tau \rightarrow \mu \nu \nu)$

$$\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004$$

- ★ Demonstrates the weak charged current is the same for all leptonic vertices



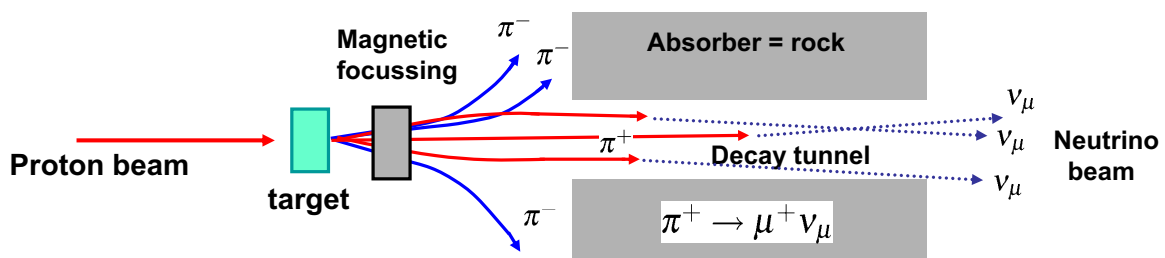
\Rightarrow **Charged Current Lepton Universality**

Neutrino Scattering

- In handout 6 considered **electron-proton** Deep Inelastic Scattering where a virtual photon is used to probe nucleon structure
- Can also consider the weak interaction equivalent: **Neutrino Deep Inelastic Scattering** where a virtual W-boson probes the structure of nucleons
 - \Rightarrow additional information about parton structure functions
 - + provides a good example of calculations of weak interaction cross sections.

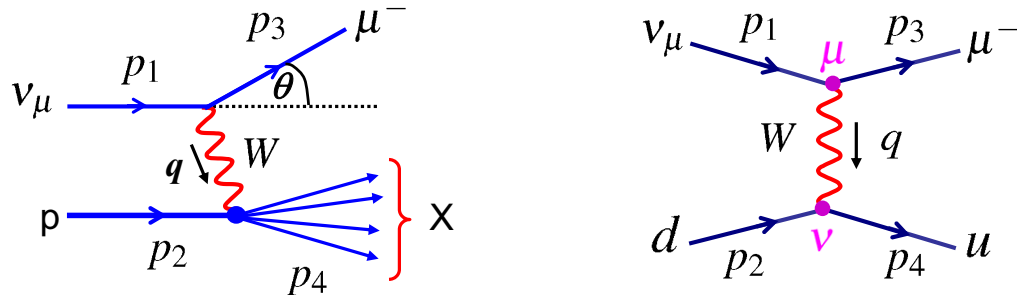
★ Neutrino Beams:

- Smash high energy protons into a fixed target \Rightarrow hadrons
- Focus positive pions/kaons
- Allow them to decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ + $K^+ \rightarrow \mu^+ \nu_\mu$ ($BR \approx 64\%$)
- Gives a beam of “collimated” ν_μ
- Focus negative pions/kaons to give beam of $\bar{\nu}_\mu$



Neutrino-Quark Scattering

★ For ν_μ -proton Deep Inelastic Scattering the underlying process is $\nu_\mu d \rightarrow \mu^- u$



★ In the limit $q^2 \ll m_W^2$ the W-boson propagator is $\approx ig_{\mu\nu}/m_W^2$

• The Feynman rules give:

$$-iM_{fi} = \left[-i \frac{g_W}{\sqrt{2}} \bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma^5) u(p_1) \right] \frac{ig_{\mu\nu}}{m_W^2} \left[-i \frac{g_W}{\sqrt{2}} \bar{u}(p_4) \frac{1}{2} \gamma^\nu (1 - \gamma^5) u(p_2) \right]$$

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}(p_3) \gamma^\mu \frac{1}{2} (1 - \gamma^5) u(p_1) \right] \left[\bar{u}(p_4) \frac{1}{2} \gamma^\nu (1 - \gamma^5) u(p_2) \right]$$

• Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected

• In this limit the helicity states are equivalent to the chiral states and

$$\frac{1}{2} (1 - \gamma^5) u_\uparrow(p_1) = 0 \quad \frac{1}{2} (1 - \gamma^5) u_\downarrow(p_1) = u_\downarrow(p_1)$$

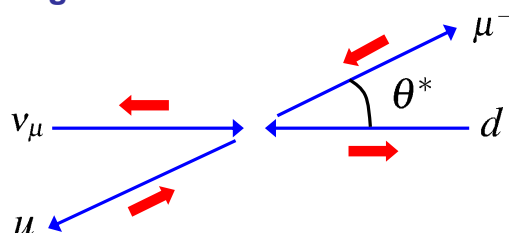
$$\Rightarrow M_{fi} = 0 \quad \text{for } u_\uparrow(p_1) \text{ and } u_\uparrow(p_2)$$

• Since the weak interaction “conserves the helicity”, the only helicity combination where the matrix element is non-zero is

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}_\downarrow(p_3) \gamma^\mu u_\downarrow(p_1) \right] \left[\bar{u}_\downarrow(p_4) \gamma^\nu u_\downarrow(p_2) \right]$$

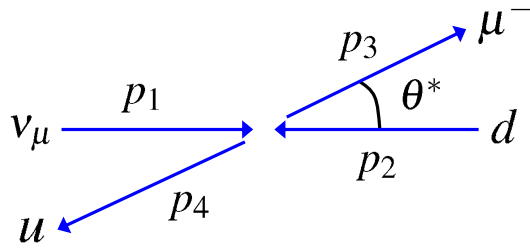
NOTE: we could have written this down straight away as in the ultra-relativistic limit only **LH helicity particle** states participate in the weak interaction.

★ Consider the scattering in the C.o.M frame



Evaluation of Neutrino-Quark Scattering ME

- Go through the calculation in gory detail (fortunately only one helicity combination)
- In CMS frame, neglecting particle masses:



$$\begin{aligned} p_1 &= (E, 0, 0, E), \\ p_2 &= (E, 0, 0, -E) \\ p_3 &= (E, E \sin \theta^*, 0, E \cos \theta^*) \\ p_4 &= (E, -E \sin \theta^*, 0, -E \cos \theta^*) \end{aligned}$$

- Dealing with LH helicity particle spinors. From handout 3 (p.80), for a massless particle travelling in direction (θ, ϕ) :

$$u_{\downarrow} = \sqrt{E} \begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \quad c = \cos \frac{\theta}{2}; \quad s = \sin \frac{\theta}{2}$$

- Here $(\theta_1, \phi_1) = (0, 0)$; $(\theta_2, \phi_2) = (\pi, 0)$; $(\theta_3, \phi_3) = (\theta^*, 0)$; $(\theta_4, \phi_4) = (\pi - \theta^*, \pi)$ giving:

$$u_{\downarrow}(p_1) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}; \quad u_{\downarrow}(p_2) = \sqrt{E} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad u_{\downarrow}(p_3) = \sqrt{E} \begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}; \quad u_{\downarrow}(p_4) = \sqrt{E} \begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

- To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} [\bar{u}_{\downarrow}(p_3) \gamma^{\mu} u_{\downarrow}(p_1)] [\bar{u}_{\downarrow}(p_4) \gamma^{\nu} u_{\downarrow}(p_2)]$$

need to evaluate two terms of form

$$\begin{aligned} \bar{\psi} \gamma^0 \phi &= \psi^{\dagger} \gamma^0 \gamma^0 \phi = \psi_1^* \phi_1 + \psi_2^* \phi_2 + \psi_3^* \phi_3 + \psi_4^* \phi_4 \\ \bar{\psi} \gamma^1 \phi &= \psi^{\dagger} \gamma^0 \gamma^1 \phi = \psi_1^* \phi_4 + \psi_2^* \phi_3 + \psi_3^* \phi_2 + \psi_4^* \phi_1 \\ \bar{\psi} \gamma^2 \phi &= \psi^{\dagger} \gamma^0 \gamma^2 \phi = -i(\psi_1^* \phi_4 - \psi_2^* \phi_3 + \psi_3^* \phi_2 - \psi_4^* \phi_1) \\ \bar{\psi} \gamma^3 \phi &= \psi^{\dagger} \gamma^0 \gamma^3 \phi = \psi_1^* \phi_3 - \psi_2^* \phi_4 + \psi_3^* \phi_1 - \psi_4^* \phi_2 \end{aligned}$$

- Using

$$u_{\downarrow}(p_1) = \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}; \quad u_{\downarrow}(p_2) = \sqrt{E} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad u_{\downarrow}(p_3) = \sqrt{E} \begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}; \quad u_{\downarrow}(p_4) = \sqrt{E} \begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

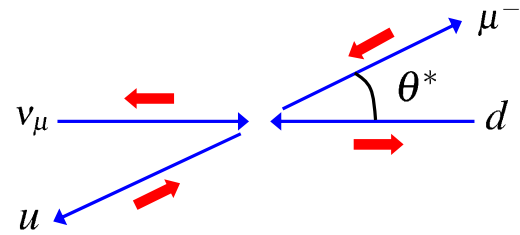
$$\begin{aligned} \bar{u}_{\downarrow}(p_3) \gamma^{\mu} u_{\downarrow}(p_1) &= 2E(c, s, -is, c) \\ \bar{u}_{\downarrow}(p_4) \gamma^{\nu} u_{\downarrow}(p_2) &= 2E(c, -s, -is, -c) \end{aligned}$$

$$\begin{aligned} \bar{u}_{\downarrow}(p_3) \gamma^{\mu} u_{\downarrow}(p_1) \bar{u}_{\downarrow}(p_4) \gamma^{\nu} u_{\downarrow}(p_2) &= 4E^2(c^2 + s^2 + s^2 + c^2) = \frac{g_W^2 \hat{s}}{m_W^2} \quad \hat{s} = (2E)^2 \end{aligned}$$

★ Note the Matrix Element is isotropic

$$M_{fi} = \frac{g_W^2}{m_W^2} \hat{s}$$

we could have anticipated this since the helicity combination (spins anti-parallel) has $S_z = 0 \rightarrow$ no preferred polar angle



★ As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination (LL→LL) and **only 2 possible initial state combinations** (the neutrino is always produced in a LH helicity state)

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2} \cdot \left| \frac{g_W^2}{m_W^2} \hat{s} \right|^2$$

The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction

★ From handout 1, in the extreme relativistic limit, the cross section for any 2→2 body scattering process is

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle = \frac{1}{64\pi \hat{s}} \frac{1}{2} \left(\frac{g_W^2 \hat{s}}{m_W^2} \right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2} \right)^2 \hat{s}$$

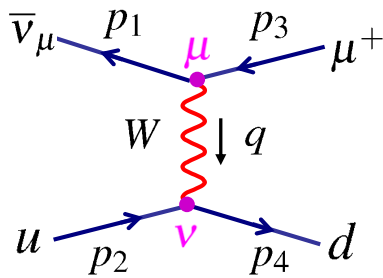
using $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} \rightarrow \boxed{\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}}$

★ Integrating this isotropic distribution over $d\Omega^*$

$$\rightarrow \boxed{\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}} \quad (1)$$

• cross section is a Lorentz invariant quantity so this is valid in any frame

Antineutrino-Quark Scattering



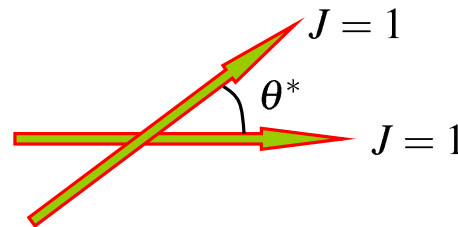
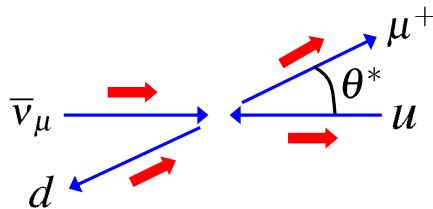
- In the ultra-relativistic limit, the charged-current interaction matrix element is:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{v}(p_1) \gamma^\mu \frac{1}{2}(1 - \gamma^5) v(p_3) \right] \left[\bar{u}(p_4) \gamma^\nu \frac{1}{2}(1 - \gamma^5) u(p_2) \right]$$

- ★ In the extreme relativistic limit only **LH Helicity particles** and **RH Helicity anti-particles** participate in the charged current weak interaction:

$$\Rightarrow M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{v}_\uparrow(p_1) \gamma^\mu v_\uparrow(p_3) \right] \left[\bar{u}_\downarrow(p_4) \gamma^\nu u_\downarrow(p_2) \right]$$

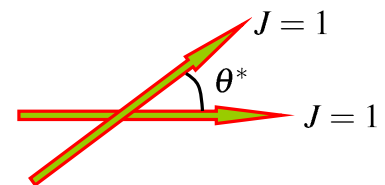
- ★ In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum 1 state



- ★ In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*} \frac{1}{4} (1 + \cos \theta^*)^2$$

- The factor $\frac{1}{4} (1 + \cos \theta^*)^2$ can be understood in terms of the overlap of the initial and final angular momentum wave-functions



- ★ In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos \theta^*)^2 \hat{s}$$

- ★ Integrating over solid angle:

$$\int (1 + \cos \theta^*)^2 d\Omega^* = \int (1 + \cos \theta^*)^2 d(\cos \theta^*) d\phi = 2\pi \int_{-1}^{+1} (1 + \cos \theta^*)^2 d(\cos \theta^*) = \frac{16\pi}{3}$$



$$\sigma_{\bar{\nu}q} = \frac{G_F^2 \hat{s}}{3\pi}$$

- ★ This is a factor three smaller than the neutrino quark cross-section

$$\frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}$$

(Anti)neutrino-(Anti)quark Scattering

- Non-zero anti-quark component to the nucleon \Rightarrow also consider scattering from \bar{q}
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include LH particles and RH anti-particles

$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}$	$\frac{d\sigma_{\bar{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos \theta^*)^2 \hat{s}$	$\frac{d\sigma_{\nu \bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos \theta^*)^2 \hat{s}$	$\frac{d\sigma_{\bar{\nu} \bar{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}$
$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}$	$\sigma_{\bar{\nu} q} = \frac{G_F^2 \hat{s}}{3\pi}$	$\sigma_{\nu \bar{q}} = \frac{G_F^2 \hat{s}}{3\pi}$	$\sigma_{\bar{\nu} \bar{q}} = \frac{G_F^2 \hat{s}}{\pi}$

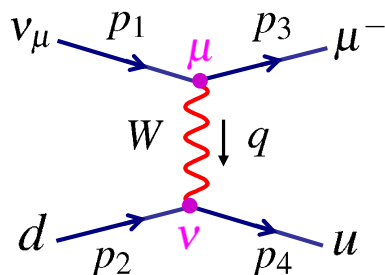
Prof. M.A. Thomson

Michaelmas 2009

319

Differential Cross Section $d\sigma/dy$

- ★ Derived differential neutrino scattering cross sections in C.o.M frame, can convert to Lorentz invariant form



- As for DIS use Lorentz invariant

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$$

- In relativistic limit y can be expressed in terms of the C.o.M. scattering angle

$$y = \frac{1}{2} (1 - \cos \theta^*)$$

- In lab. frame

$$y = 1 - \frac{E_3}{E_1}$$

- ★ Convert from $\frac{d\sigma}{d\Omega^*} \rightarrow \frac{d\sigma}{dy}$ using

$$\frac{d\sigma}{dy} = \left| \frac{d\cos \theta^*}{dy} \right| \frac{d\sigma}{d\cos \theta^*} = \left| \frac{d\cos \theta^*}{dy} \right| 2\pi \frac{d\sigma}{d\Omega^*} = 4\pi \frac{d\sigma}{d\Omega^*}$$

- Already calculated (1)

$$\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}$$

- Hence:

$$\boxed{\frac{d\sigma_{\nu q}}{dy} = \frac{d\sigma_{\bar{\nu} \bar{q}}}{dy} = \frac{G_F^2}{\pi} \hat{s}}$$

Prof. M.A. Thomson

Michaelmas 2009

320

and $\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2} (1 + \cos \theta^*)^2 \hat{s}$

becomes $\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{4\pi} (1 + \cos \theta^*)^2 \hat{s}$

from $y = \frac{1}{2}(1 - \cos \theta^*) \rightarrow 1 - y = \frac{1}{2}(1 + \cos \theta^*)$

and hence $\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{\pi} (1 - y)^2 \hat{s}$

★ For comparison, the electro-magnetic $e^\pm q \rightarrow e^\pm q$ cross section is:

QED $\frac{d\sigma_{e^\pm q}}{dy} = \frac{2\pi\alpha^2}{q^4} e_q^2 [1 + (1 - y)^2] \hat{s}$

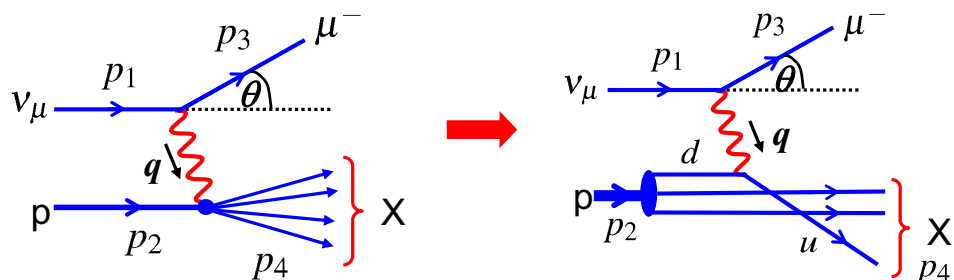
DIFFERENCES:

Interaction
+propagator

Helicity
Structure

WEAK $\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{\pi} (1 - y)^2 \hat{s}$

Parton Model For Neutrino Deep Inelastic Scattering



Scattering from a proton
with structure functions

Scattering from a point-like
quark within the proton

★ Neutrino-proton scattering can occur via scattering from a down-quark or from an anti-up quark

- In the parton model, number of down quarks within the proton in the momentum fraction range $x \rightarrow x + dx$ is $d^P(x)dx$. Their contribution to the neutrino scattering cross-section is obtained by multiplying by the $\nu_\mu d \rightarrow \mu^- u$ cross-section derived previously

$$\frac{d\sigma^{\nu P}}{dy} = \frac{G_F^2}{\pi} \hat{s} d^P(x) dx$$

where \hat{s} is the centre-of-mass energy of the $\nu_\mu d$

- Similarly for the \bar{u} contribution

$$\frac{d\sigma^{\nu p}}{dy} = \frac{G_F^2}{\pi} \hat{s}(1-y)^2 \bar{u}^p(x) dx$$

- ★ Summing the two contributions and using $\hat{s} = xs$

$$\Rightarrow \frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} sx [d^p(x) + (1-y)^2 \bar{u}^p(x)]$$

- ★ The anti-neutrino proton differential cross section can be obtained in the same manner:

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2}{\pi} sx [(1-y)^2 u^p(x) + \bar{d}^p(x)]$$

- ★ For (anti)neutrino – neutron scattering:

$$\frac{d^2\sigma^{\nu n}}{dx dy} = \frac{G_F^2}{\pi} sx [d^n(x) + (1-y)^2 \bar{u}^n(x)]$$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dx dy} = \frac{G_F^2}{\pi} sx [(1-y)^2 u^n(x) + \bar{d}^n(x)]$$

- As before, define neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^p(x) = d^n(x); \quad d(x) \equiv d^p(x) = u^n(x)$$

$$\bar{u}(x) \equiv \bar{u}^p(x) = \bar{d}^n(x); \quad \bar{d}(x) \equiv \bar{d}^p(x) = \bar{u}^n(x)$$

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} sx [d(x) + (1-y)^2 \bar{u}(x)] \quad (2)$$

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2}{\pi} sx [(1-y)^2 u(x) + \bar{d}(x)] \quad (3)$$

$$\frac{d^2\sigma^{\nu n}}{dx dy} = \frac{G_F^2}{\pi} sx [u(x) + (1-y)^2 \bar{d}(x)] \quad (4)$$

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2}{\pi} sx [(1-y)^2 d(x) + \bar{u}(x)] \quad (5)$$

- ★ Because neutrino cross sections are very small, need massive detectors. These are usually made of Iron, hence, experimentally measure a combination of proton/neutron scattering cross sections

- ★ For an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon:

$$\frac{d^2\sigma^{vN}}{dx dy} = \frac{1}{2} \left(\frac{d^2\sigma^{vp}}{dx dy} + \frac{d^2\sigma^{vn}}{dx dy} \right)$$



$$\frac{d^2\sigma^{vN}}{dx dy} = \frac{G_F^2}{2\pi} s x [u(x) + d(x) + (1-y)^2(\bar{u}(x) + \bar{d}(x))]$$

- Integrate over momentum fraction x

$$\frac{d\sigma^{vN}}{dy} = \frac{G_F^2}{2\pi} s [f_q + (1-y)^2 f_{\bar{q}}] \quad (6)$$

where f_q and $f_{\bar{q}}$ are the total momentum fractions carried by the quarks and by the anti-quarks within a nucleon

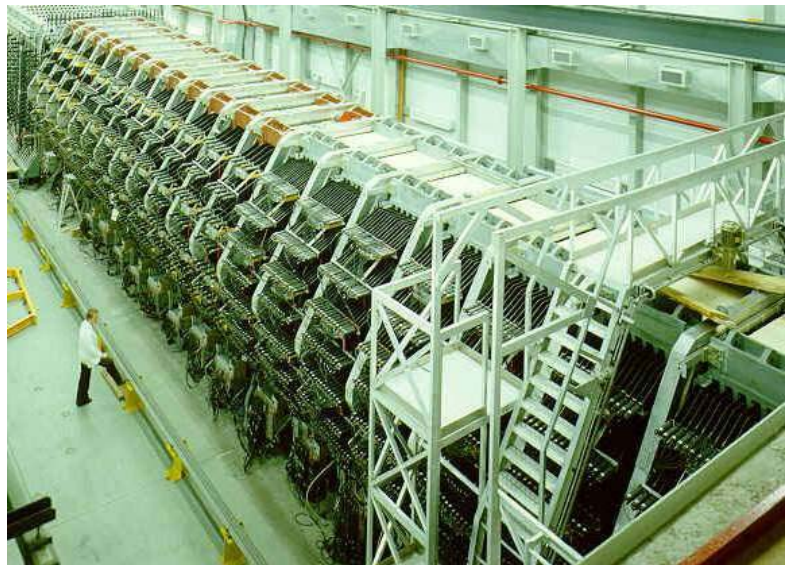
$$f_q \equiv f_d + f_u = \int_0^1 x [u(x) + d(x)] dx; \quad f_{\bar{q}} \equiv f_{\bar{d}} + f_{\bar{u}} = \int_0^1 x [\bar{u}(x) + \bar{d}(x)] dx$$

- Similarly

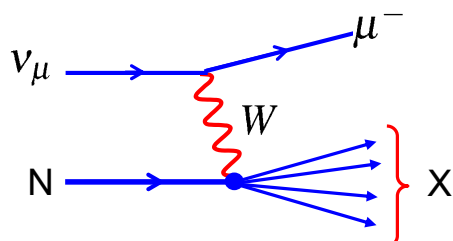
$$\frac{d\sigma^{\bar{v}N}}{dy} = \frac{G_F^2}{2\pi} s [(1-y)^2 f_q + f_{\bar{q}}] \quad (7)$$

e.g. CDHS Experiment (CERN 1976-1984)

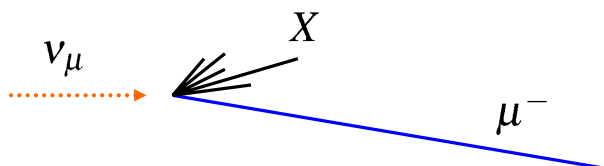
- 1250 tons
- Magnetized iron modules
- Separated by drift chambers



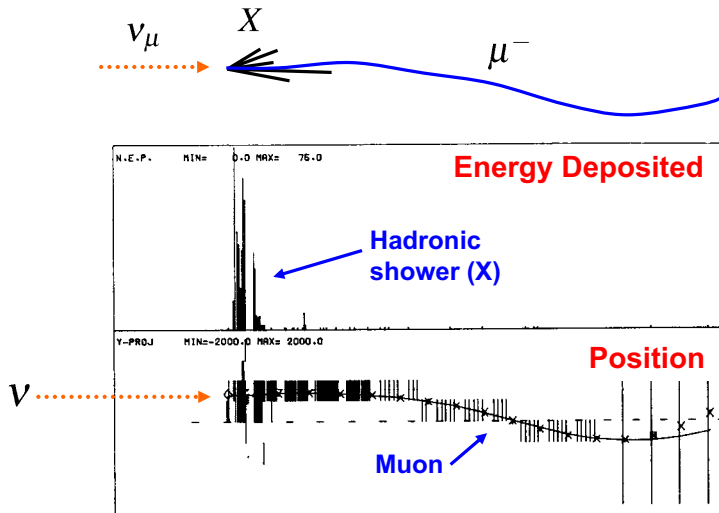
Study Neutrino Deep Inelastic Scattering



Experimental Signature:



Example Event:



• Measure energy of X
 E_X

• Measure muon momentum from curvature in B-field
 E_μ

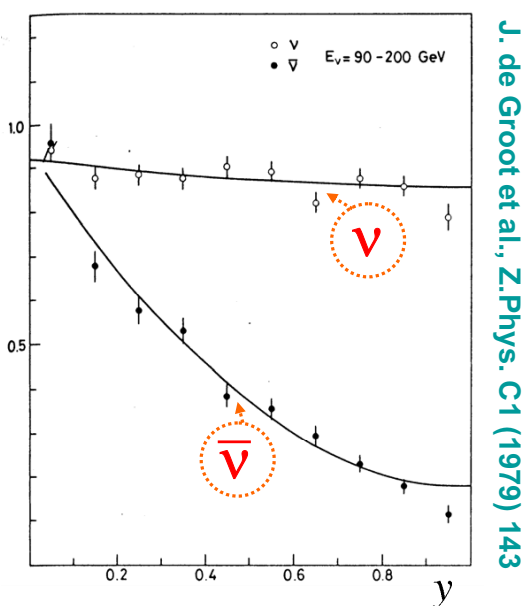
★ For each event can determine neutrino energy and y !

$$E_\nu = E_X + E_\mu$$

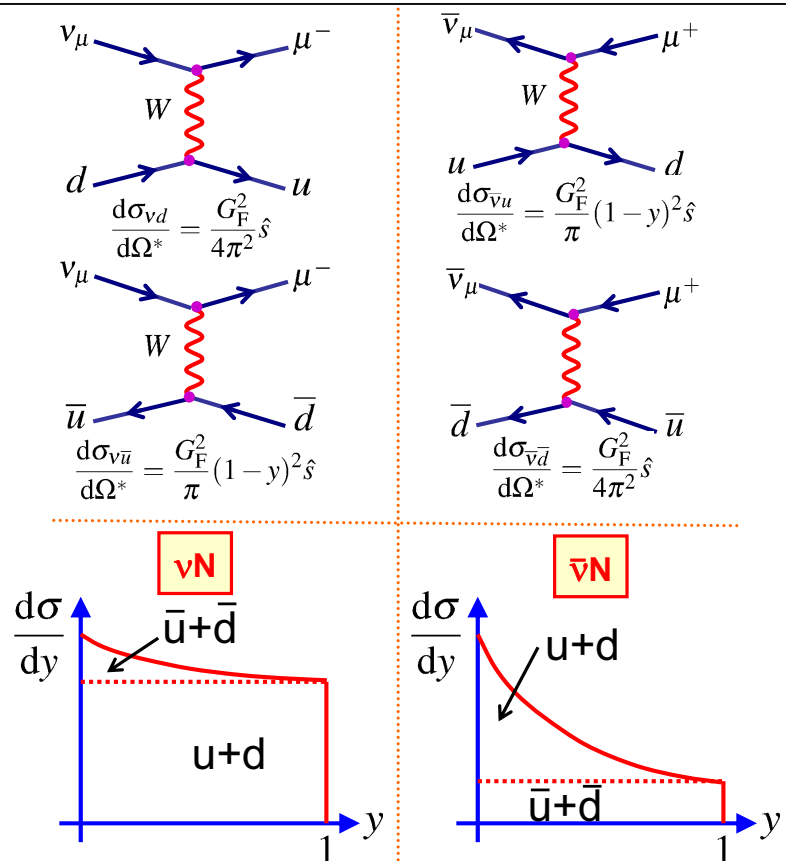
$$E_\mu = (1 - y)E_\nu \rightarrow y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

Measured y Distribution

• CDHS measured y distribution



• Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering



Measured Total Cross Sections

- ★ Integrating the expressions for $\frac{d\sigma}{dy}$ (equations (6) and (7))

$$\sigma^{\nu N} = \frac{G_F^2 s}{2\pi} \left[f_q + \frac{1}{3} f_{\bar{q}} \right]$$

$$\sigma^{\bar{\nu} N} = \frac{G_F^2 s}{2\pi} \left[\frac{1}{3} f_q + f_{\bar{q}} \right]$$

$$\begin{array}{ccc} \nu & \longrightarrow & p \\ (E_\nu, 0, 0, +E_\nu) & (m_p, 0, 0, 0) & \end{array} \quad s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$$

→ DIS cross section \propto lab. frame neutrino energy

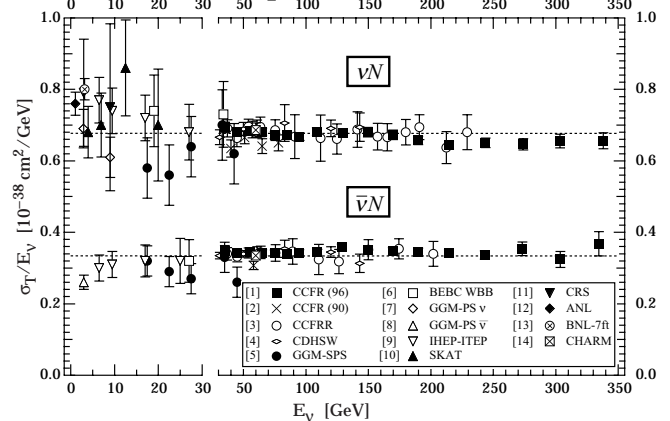
- ★ Measure cross sections can be used to determine fraction of protons momentum carried by quarks, f_q , and fraction carried by anti-quarks, $f_{\bar{q}}$

- Find: $f_q \approx 0.41$; $f_{\bar{q}} \approx 0.08$
- ~50% of momentum carried by gluons (which don't interact with virtual W boson)
- If no anti-quarks in nucleons expect

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = 3$$

- Including anti-quarks

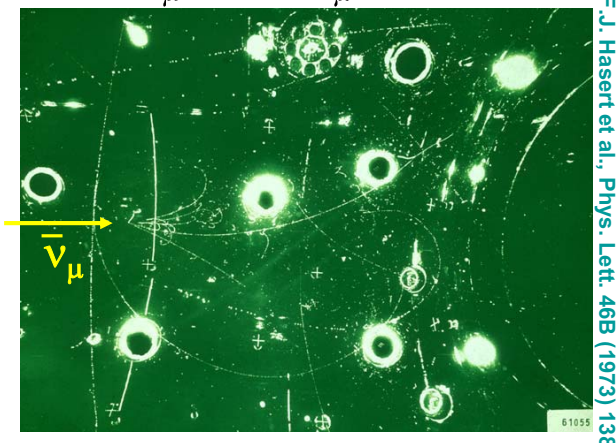
$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} \approx 2$$



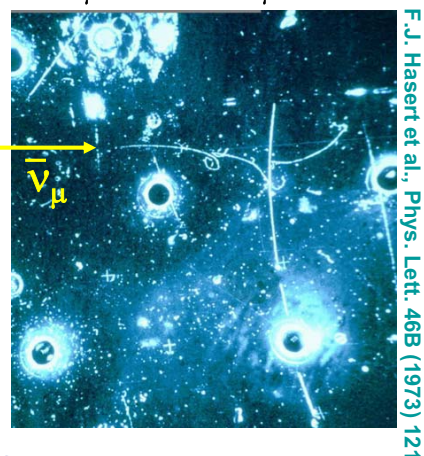
Weak Neutral Current

- ★ Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973. Interaction of muon neutrinos produce a final state muon

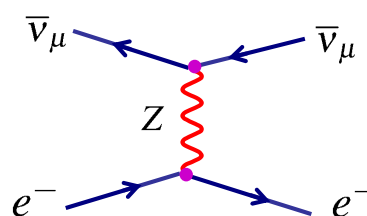
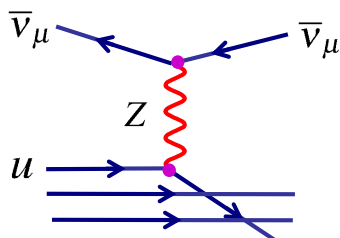
$$\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + \text{hadrons}$$



$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$



- ★ Cannot be due to W exchange - first evidence for Z boson



Summary

- ★ **Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections**
- ★ **Neutrino – nucleon scattering yields extra information about parton distributions functions:**
 - ν couples to d and \bar{u} ; $\bar{\nu}$ couples to u and \bar{d}
 - ➔ investigate flavour content of nucleon
 - can measure anti-quark content of nucleon
 - $\nu\bar{q}$ suppressed by factor $(1-y)^2$ compared with νq
 - $\bar{\nu}q$ suppressed by factor $(1-y)^2$ compared with $\bar{\nu}\bar{q}$
- ★ **Further aspects of neutrino deep-inelastic scattering (expressed in general structure functions) are covered in Appendix II**
- ★ **Finally observe that neutrinos interact via weak neutral currents (NC)**

Appendix I

- **For the adjoint spinors $\bar{u} = u^\dagger \gamma^0$ consider**

$$\frac{1}{2}(1 - \gamma^5)u = [\frac{1}{2}(1 - \gamma^5)u]^\dagger \gamma^0 = u^\dagger \frac{1}{2}(1 - \gamma^5)\gamma^0 = u^\dagger \gamma^0 \frac{1}{2}(1 + \gamma^5) = \bar{u} \frac{1}{2}(1 + \gamma^5)$$

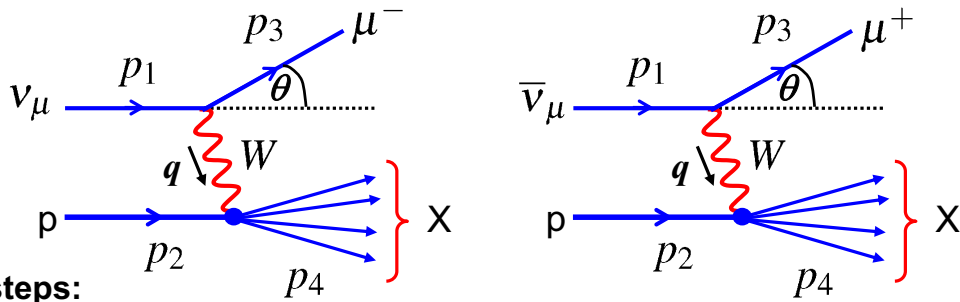
$$\frac{1}{2}(1 - \gamma^5)u_\uparrow = 0 \quad \rightarrow \quad \bar{u} \frac{1}{2}(1 + \gamma^5) = 0$$

Using the fact that γ^5 and γ^μ anti-commute can rewrite ME:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}(p_3) \frac{1}{2}(1 + \gamma^5) \gamma^\mu u(p_1) \right] \left[\bar{u}(p_4) \frac{1}{2}(1 + \gamma^5) \gamma^\nu u(p_2) \right]$$

$$\rightarrow M_{fi} = 0 \quad \text{for} \quad \bar{u}_\uparrow(p_3) \quad \text{and} \quad \bar{u}_\uparrow(p_4)$$

Appendix II: Deep-Inelastic Neutrino Scattering



Two steps:

- First write down most general cross section in terms of structure functions
- Then evaluate expressions in the quark-parton model

QED Revisited

- ★ In the limit $s \gg M^2$ the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written (Eq. 2 of handout 6)

$$\frac{d^2\sigma_{e^\pm p}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

- For neutrino scattering typically measure the energy of the produced muon $E_\mu = E_\nu(1-y)$ and differential cross-sections expressed in terms of $dx dy$
- Using $Q^2 = (s - M^2)xy \approx sxy \Rightarrow \frac{d^2\sigma}{dx dy} = \left| \frac{dQ^2}{dy} \right| \frac{d^2\sigma}{dx dQ^2} = sx \frac{d^2\sigma}{dx dQ^2}$

- ♦ In the limit $s \gg M^2$ the general Electro-magnetic DIS cross section can be written

$$\frac{d^2\sigma^{e^\pm p}}{dx dy} = \frac{4\pi\alpha^2 s}{Q^4} \left[(1-y) F_2(x, Q^2) + y^2 x F_1(x, Q^2) \right]$$

- NOTE: This is the most general Lorentz Invariant **parity conserving** expression

- ★ For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function $F_3(x, Q^2)$

$$\nu_\mu p \rightarrow \mu^- X \quad \frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[(1-y) F_2^{\nu p}(x, Q^2) + y^2 x F_1^{\nu p}(x, Q^2) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x, Q^2) \right]$$

- For anti-neutrino scattering new structure function enters with opposite sign

$$\bar{\nu}_\mu p \rightarrow \mu^+ X \quad \frac{d^2\sigma^{\bar{\nu} p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[(1-y) F_2^{\bar{\nu} p}(x, Q^2) + y^2 x F_1^{\bar{\nu} p}(x, Q^2) - y \left(1 - \frac{y}{2}\right) x F_3^{\bar{\nu} p}(x, Q^2) \right]$$

- Similarly for neutrino-neutron scattering

$$\nu_\mu n \rightarrow \mu^- X \quad \frac{d^2\sigma^{\nu n}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[(1-y) F_2^{\nu n}(x, Q^2) + y^2 x F_1^{\nu n}(x, Q^2) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu n}(x, Q^2) \right]$$

$$\bar{\nu}_\mu n \rightarrow \mu^+ X \quad \frac{d^2\sigma^{\bar{\nu} n}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[(1-y) F_2^{\bar{\nu} n}(x, Q^2) + y^2 x F_1^{\bar{\nu} n}(x, Q^2) - y \left(1 - \frac{y}{2}\right) x F_3^{\bar{\nu} n}(x, Q^2) \right]$$

Neutrino Interaction Structure Functions

★ In terms of the parton distribution functions we found (2) :

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2}{\pi} s x [d(x) + (1-y)^2 \bar{u}(x)]$$

• Compare coefficients of y with the general Lorentz Invariant form (p.321) and assume Bjorken scaling, i.e. $F(x, Q^2) \rightarrow F(x)$

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[(1-y) F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x) \right]$$

• Re-writing (2) $\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi} [2xd(x) + 2x\bar{u}(x) - 4xy\bar{u}(x) + 2xy^2\bar{u}(x)]$

and equating powers of y

$$\left. \begin{aligned} 2xd + 2x\bar{u} &= F_2 \\ -4x\bar{u} &= -F_2 + xF_3 \\ 2\bar{u} &= F_1 - xF_3/2 \end{aligned} \right\}$$

gives:

$$\boxed{\begin{aligned} F_2^{\nu p} &= 2xF_1^{\nu p} = 2x[d(x) + \bar{u}(x)] \\ xF_3^{\nu p} &= 2x[d(x) - \bar{u}(x)] \end{aligned}}$$

NOTE: again we get the **Callan-Gross relation** $F_2 = 2xF_1$

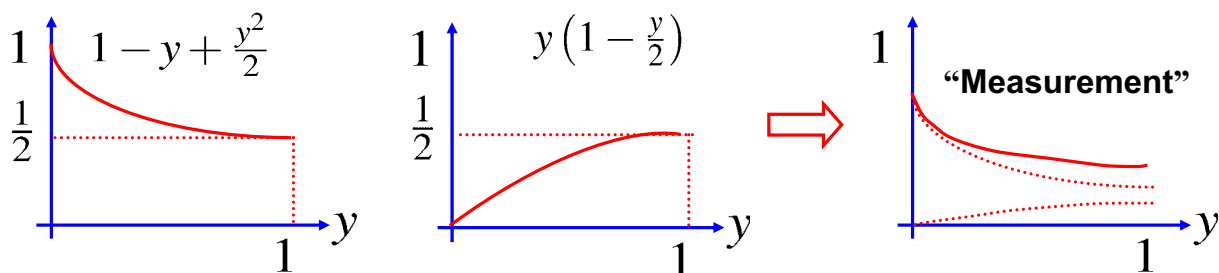
No surprise, underlying process is scattering from point-like spin-1/2 quarks

★ Substituting back in to expression for differential cross section:

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_F^2 s}{2\pi} \left[\left(1 - y + \frac{y^2}{2}\right) F_2^{\nu p}(x) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x) \right]$$

★ Experimentally measure F_2 and F_3 from y distributions at fixed x

♦ Different y dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured



★ Then use $F_2^{\nu p} = 2x[d(x) + \bar{u}(x)]$ and $F_3^{\nu p} = 2[d(x) - \bar{u}(x)]$

➡ Determine $d(x)$ and $\bar{u}(x)$ separately

★ Neutrino experiments require large detectors (often iron) i.e. isoscalar target

$$F_2^{\nu N} = 2xF_1^{\nu N} = \frac{1}{2} (F_2^{\nu p} + F_2^{\nu n}) = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

$$xF_3^{\nu N} = \frac{1}{2} (xF_3^{\nu p} + xF_3^{\nu n}) = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$$

★ For electron – nucleon scattering:

$$F_2^{ep} = 2xF_1^{ep} = x[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)]$$

$$F_2^{en} = 2xF_1^{en} = x[\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)]$$

• For an isoscalar target

$$F_2^{eN} = \frac{1}{2} (F_2^{ep} + F_2^{en}) = \frac{5}{18}x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

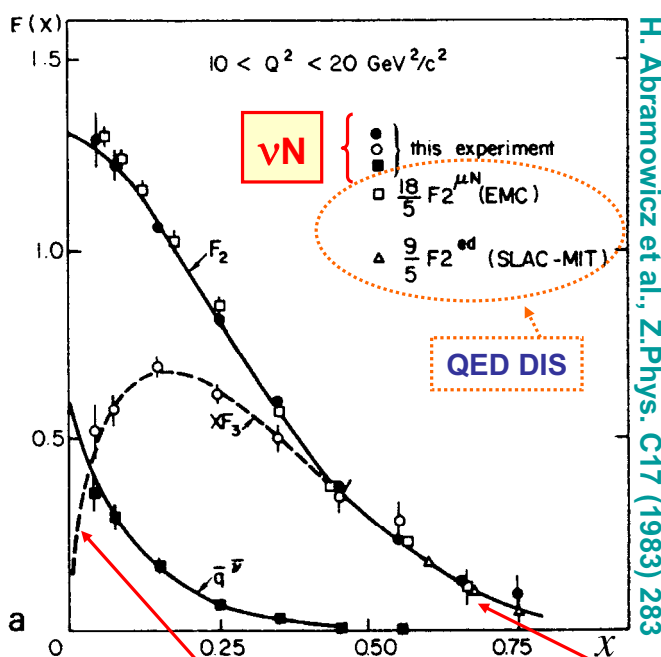
$$\rightarrow F_2^{\nu N} = \frac{18}{5}F_2^{eN}$$

• Note that the factor $\frac{5}{18} = \frac{1}{2} (q_u^2 + q_d^2)$ and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

Experiment: 0.29 ± 0.02

Measurements of $F_2(x)$ and $F_3(x)$

• CDHS Experiment $\nu_\mu + \text{Fe} \rightarrow \mu^- + X$



$$F_2^{\nu N} = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

$$xF_3^{\nu N} = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$$

$$\rightarrow F_2^{\nu N} - xF_3^{\nu N} = 2x[\bar{u} + \bar{d}]$$

★ Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions

Sea dominates so expect xF_3 to go to zero as $q(x) = \bar{q}(x)$

Sea contribution goes to zero

Valence Contribution

★ Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$

$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) = S(x)$$

$$\bar{d}(x) = \bar{d}_S(x) = S(x)$$

→ $F_3^{vN} = [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)] = u_V(x) + d_V(x)$

→ $\int_0^1 F_3^{vN}(x) dx = \int_0^1 (u_V(x) + d_V(x)) dx = N_u^V + N_d^V$

★ Area under measured function $F_3^{vN}(x)$ gives a measurement of the total number of valence quarks in a nucleon !

expect $\int_0^1 F_3^{vN}(x) dx = 3$ “Gross – Llewellyn-Smith sum rule”

Experiment: 3.0 ± 0.2

•Note: $F_2^{\bar{v}p} = F_2^{vn}$; $F_2^{\bar{v}n} = F_2^{vp}$; $F_3^{\bar{v}p} = F_3^{vn}$; $F_3^{\bar{v}n} = F_3^{vp}$ and anti-neutrino structure functions contain same pdf information