More on symmetries   Lectures in Week April 16, 2012

Translation & rotation operators

$$T_a \psi(x) = \psi(x+\Delta a) = \psi(x) + (\Delta a)\frac{d\psi}{dx}\,, \quad \text{at small } a$$

$$T_a \simeq 1 + (\Delta a)\frac{\partial}{\partial x} = 1 + i(\Delta a)\, p_x/\hbar\,, \qquad p_x = \frac{1}{i}\frac{\partial}{\partial x}$$

For a finite translation a, we can make n steps in succession

$a = n(\Delta a)$, and take $n \to \infty$

$$T_a = \lim_{n\to\infty}\left(1 + \frac{i(\Delta a)\, p_x}{\hbar}\right)^n = e^{ipa/\hbar}$$

Similarly, the generator of an infinitesimal rotation about some axis may be written

$$R = 1 + \Delta\varphi \frac{\partial}{\partial \varphi}$$

and the operator for the z-component of angular momentum is

$$J_z = -i\hbar \frac{\partial}{\partial \phi}$$

so that $R = 1 + i J_z \Delta\varphi/\hbar$ for small $\Delta\varphi$

or $R = e^{iJ_z\varphi/\hbar}$ for a finite φ.

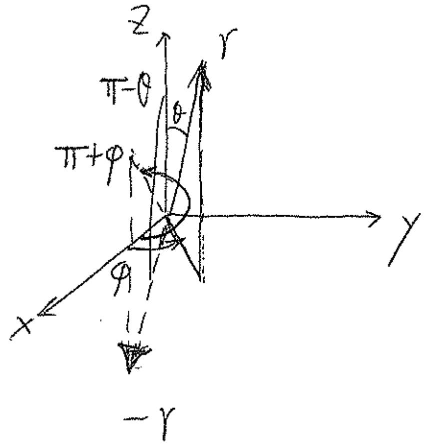
parity operator

$$P\psi(\vec{r}) = \psi(-\vec{r})$$

Since $P^2\psi(\vec{r}) = \psi(\vec{r})$, $\therefore$ $P = \pm 1$ — eigenvales of P.

under ~~par~~ inversion, $\vec{r} \to -\vec{r}$

$$(r, \theta, \varphi) \longrightarrow (r, \pi-\theta, \varphi+\pi).$$



Wavefunction in spherical coordinator

$$\psi(r,\theta,\varphi) = \chi(r)\, Y_{lm}(\theta,\varphi)$$

$$P\, Y_{lm}(\theta,\varphi) = Y_{lm}(\pi-\theta, \pi+\varphi) = (-1)^l\, Y_{lm}(\theta,\varphi)$$

$$\because \quad Y_{lm}(\theta,\varphi) \propto P_l^m(\cos\theta)\, e^{im\varphi}$$

parity is a multiplicative quantum number, and is conserved in strong & e.m. interaction.

Intrinsic parity

for bayron like proton, neutron, bayron number is conserved, so it is a convention to assign their parity $+1$. However anti ~~particle~~ fermion has opposite intrinsic parity of its ~~particle~~ fermion, as a result of Dirac theory, varified in experiments.

for instance, $\bar{p}$ and p have opposite intrinsic parity, and e^+ and e^- have opposite intrinsic parity too.

quark, we may assign $P_q = +1$, hence $P_{proton} = P_{neutron} = +1$

Note that proton & neutron consist of 3 quarks each and the total orbital angular momentum $= 0$, hence the relative motion does not alter the parity.

anti quark has intrinsic parity -1 : $P_{\bar{q}} = -1$.

We then can discuss pions intrinsic parity.

$\pi^+ = |u\bar{d}\rangle$, $L=0$. So the intrinsic parity is $P_{\pi^\pm} = -1$.

Similarly, π^- and π^0 have intrinsic parity -1 too.

These particles have zero angular momentum but change sign under inversion, called pseudo scalar.

We now discuss how the spin & ~~pari~~ intrinsic parity initially determined in experiments.

Spin of the pion π^+.

$$p + p \rightleftharpoons \pi^+ + d$$

This is a forward & backward scattering process. If the reaction is balanced, the forward & backward scattering matrix elements will be the same, S_π, S_d, S_p : spins of π, d and p.

$$\sigma_{pp\to\pi^+ d} \propto (2S_\pi+1)(2S_d+1)\, p_\pi^2$$

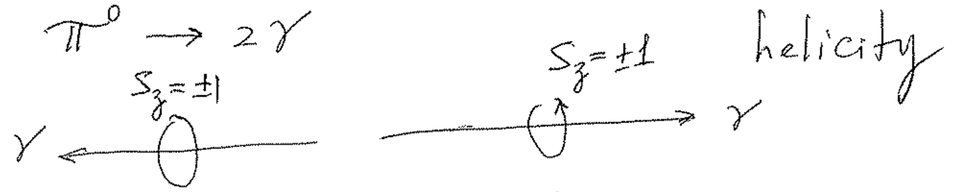
$$\sigma_{\pi^+ d\to pp} \propto \frac{1}{2}(2S_p+1)^2\, p_p^2$$

The prefactor of $\frac{1}{2}$ in the second eqn. is due to the identical particle of proton in the final state. The spin factor $(2S+1)$ is due to spin degeneracy.

We know $S_p = \frac{1}{2}$, ad deuteron has spin 1, the measuremet gave $S_\pi = 0$, (1951').

<u>Spin of π^0</u>

$$\pi^0 \rightarrow 2\gamma$$



Suppose $S_{\pi^0} = 1$, then only $S_z^{\text{total of } 2\gamma} = 0$ is possible, with Negative parity, or a sign change in interchange the 2 photons. violate boson symmetry. $\Rightarrow$ $S_{\pi^0} = 0$ or ≥ 2.

<u>parity of π^-.</u>

$$\pi^- + d \rightarrow n + n.$$

initial state: π^- ad d are in $L=0$ state. d has spin 1, π^- has spin 0. So the total angular momentum of the initial state is $J=1$.

The final state must have $J=1$, which allows the 2n state with spin singlet $S_{2n}=0$, then $L_{2n}=1$ or $S_{2n}=1$, then $L_{2n}=0$ or 2 or 1

Since the total w.f. of $n+n$ state must be antisymmetric, it requires $S_{2n} + L_{2n} = \text{even}$. Therefor $S_{2n}=1$, $L_{2n}=1$.

The parity of the finite state $n+n$ is negative, hence π^- has negative parity. The quark parity analysis is consistent with the experiment.

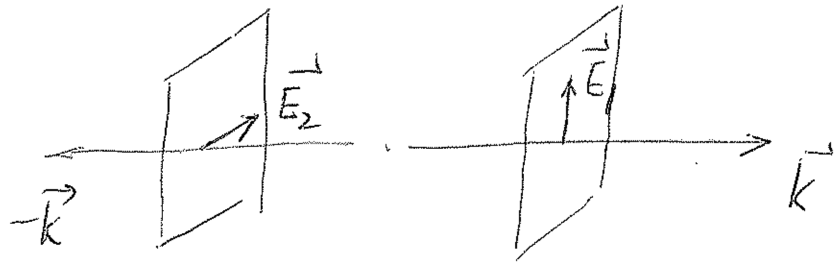
boson and its anti boson have the same intrinsic parity. For π-ions, we can understand this from the quark model.

experiment confirmation of fermion + anti-fermion parities

$$e^+ e^- \longrightarrow 2\gamma$$

The initial state has an orbital angular momentum $L=0$, like hydrogen atom. The total spin $S=0$. The parity of e^+e^- ~~depends on~~ can be determined by the parity of the final states of 2γ.

The photons are bosons and symmetric under interchange of the 2 particles.



A Photon is described by its wave vector $\vec{k}$ and the polarization $\vec{E}$, the electric field which is $\perp \vec{k}$.

With even exchange symmetry, the 2 photon wavefunction has the following forms

$$\psi_1(2\gamma) \propto \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 \propto \cos\varphi$$

$$\psi_2(2\gamma) \propto \left(\vec{\epsilon}_1 \times \vec{\epsilon}_2\right) \cdot \vec{k} \propto \sin\varphi$$

where φ is the angle between $\vec{E}_1$ and $\vec{E}_2$, and $\vec{\epsilon}_1$, $\vec{\epsilon}_2$ are the unit vector of $\vec{E}_1$ & $\vec{E}_2$.

ψ_1 has even parity since under the parity

$$\vec{E} \to -\vec{E}.$$

ψ_2 has odd parity.

The ~~amplitude~~ transition rate of the process is $\propto |\psi|^2$.

It shows odd parity.