University of Kerbala
College of Science
Department of Physics

Stage: 4th
Course Title: Math. Methods for Physics
Examiner: Mohammed Al-Kaabi
Time: 3 hours

Final Examination 2012-2013
Note: Number of questions (4) Answer (4) only

Q1. (A) The simultaneous equations representing the currents flowing in an unbalanced, three-phase, star-connected, electrical network are as follows: (6 Mark)

$$\begin{aligned} 2.4\ I_1 + 3.6\ I_2 + 4.8\ I_3 &= 1.2 \\ -3.9\ I_1 + 1.3\ I_2 - 6.5\ I_3 &= 2.6 \\ 1.7\ I_1 + 11.9\ I_2 + 8.5 I_3 &= 0 \end{aligned}$$

Using matrices, solve the equations for I_1, I_2 and I_3.

B. Find $\mathcal{L}^{-1}\left\{\frac{3s^3+s^2+12s+2}{(s-3)(s+1)^3}\right\}$. (6 Mark)

Q2. (A) The Fourier series of the function $f(\theta) = \theta^2$ in the range $-\pi < \theta < \pi$ is given by: (8 Mark)

$$f(\theta) = \theta^2 = \frac{\pi^2}{3} - 4\left(\cos\theta - \frac{1}{2^2}\cos 2\theta + \frac{1}{3^2}\cos 3\theta - \frac{1}{4^2}\cos 4\theta + \frac{1}{5^2}\cos 5\theta \ldots\right)$$

Now, let $\theta = \pi$ to show that:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

B. If $a^{ij}x_i x_j = 0$ where a_{ij} are constant $a_{ij} + a_{ji} = 0$. (4 Mark)

Q3. (A) if $\tilde{S}$ is a real antisymmetric matrix then, prove that: (6 Mark)

$$\tilde{A} = (\tilde{I} - \tilde{S})(\tilde{I} + \tilde{S})^{-1},$$

is orthogonal.

B. Determine the Laplace transforms of the following: (8 Mark)

a. $5e^{-3t}\sinh 2t$ **b.** $2e^{3t}(4\cos 2t - 5\sin 2t)$

Q4. (A) If a unitary matrix $\tilde{U}$ is written as $\tilde{A} + i\tilde{B}$, where $\tilde{A}$ and $\tilde{B}$ are Hermitian and commutate. Show the following: $\tilde{A}^2 + \tilde{B}^2 = \tilde{I}$ (6 Mark)

B. Show that the relation between the components resolved in original system (x, y) And those resolved in the new rotated system $(\acute{x}, \acute{y})$ are given by: (6 Mark)

$$\acute{x} = x\cos\emptyset + y\sin\emptyset$$
$$\acute{y} = -x\sin\emptyset + y\cos\emptyset$$

Good Luck

M. Al Kaabi
Signature of the Course Lecturer

N. ALDAHAN
Head of the Department signature

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Q1 (A)

$$D = \begin{vmatrix} 2.4 & 3.6 & 4.8 \\ -3.9 & 1.3 & -6.5 \\ 1.7 & 11.9 & 8.5 \end{vmatrix} = 2.4(11.05+77.35) - 3.6(-33.15+11.05) + 4.8(-46.41-2.2)$$
$$= 212.16 + 79.56 - 233.376 = 58.344 \Rightarrow I_1 = \text{—}$$

$$D_1 = \begin{vmatrix} 1.2 & 3.6 & 4.8 \\ 2.6 & 1.3 & -6.5 \\ 0 & 11.9 & 8.5 \end{vmatrix} = 1.2(11.05+77.35) - 3.6(22.1) + 4.8(30.94) =$$
$$= 103.87 - 79.56 + 148.512 = 172.822$$

$$I_1 = \frac{172.822}{58.344} = 2.96$$

$$D_2 = \begin{vmatrix} 2.4 & 1.2 & 4.8 \\ -3.9 & 2.6 & -6.5 \\ 1.7 & 0 & 8.5 \end{vmatrix} = 2.4(22.1) - 1.2(-33.15+11.05) + 4.8(-4.42)$$
$$= 53.04 + 26.52 - 21.216 = 58.344$$

$$I_2 = \frac{58.344}{58.344} = 1$$

$$D_3 = \begin{vmatrix} 2.4 & 3.6 & 1.2 \\ -3.9 & 1.3 & 2.6 \\ 1.7 & 11.9 & 0 \end{vmatrix} = 2.4(-4.94) - 3.6(-4.42) + 1.2(-46.41-2.21)$$
$$= -11.85 + 15.912 - 58.344 = -54.282$$
$$\Rightarrow I_3 = \frac{-54.28}{58.344} = 0.93$$

(B)

$$\mathcal{L}^{-1}\left\{ \frac{3s^3+s^2+12s+2}{(s-3)(s+1)^3} \right\} = \frac{A}{s-3} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}$$

$$= 3s^3+s^2+12s+2 = A(s+1)^3 + B(s-3)(s+1)^2 + C(s-3)(s+1) + D(s-3)$$

when $s=3$, $123 = 64A \Rightarrow A = 2$

$= \quad s=-1, \; -12 = -4D \Rightarrow D = 3$

Equating s^3 terms gives $3 = A+B$, from which, $B = 1$

then $2 = A - 3B - 3C - 3D$

i.e. $2 = 2 - 3 - 3C - 9 \Rightarrow 3C = -12 \Rightarrow C = -4$, Hence

$$\mathcal{L}^{-1}\{ \quad \} = \mathcal{L}^{-1}\left\{ \frac{2}{s-3} + \frac{1}{s+1} - \frac{4}{(s+1)^2} + \frac{3}{(s+1)^3} \right\}$$

$$= 2e^{3t} + e^{-t} - 4e^{-t}t + \frac{3}{2}e^{-t}t^2$$

Q2/ (A) $\pi^2 = \frac{\pi^2}{3} - 4\left(\cos\pi - \frac{1}{2^2}\cos 2\pi + \frac{1}{3^2}\cos 3\pi - \frac{1}{4^2}\cos 4\pi + \frac{1}{5^2}\cos 5\pi \dots\right)$

i.e.

$$\pi^2 - \frac{\pi^2}{3} = -4\left(-1 - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{5^2} - \dots\right)$$

$$\frac{2\pi^2}{3} = 4\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots\right)$$

$$\frac{\cancel{2}\pi^2}{3 * \cancel{4}} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots \Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

B) $a^{ij} x_i x_j = 0$, a_{ij} arbitrary, prove $a_{ij} + a_{ji} = 0$

$a_{ij} x^i x^j = 0$

$a_{em} x^l x^m = 0$

$$\frac{\partial}{\partial x^i}(a_{em} x^l x^m) = 0 \Rightarrow a_{em}\frac{\partial}{\partial x^i}(x^l x^m) = 0$$

$$= a_{em} x^l \frac{\partial x^m}{\partial x^i} + a_{em} x^m \frac{\partial x^l}{\partial x^i} = 0$$

$$= a_{em} x^l \delta^m_i + a_{em} x^m \delta^l_i = a_{em}\delta^m_i x^l + a_{em}\delta^l_i x^m = a_{li} x^l + a_{im} x^m$$

now differentiating it w.r.t x_j ,-

$$\frac{\partial}{\partial x_j}\frac{\partial}{\partial x^i}(a_{em} x^l x^m) = 0 \Rightarrow \frac{\partial}{\partial x_j}(a_{li} x^l + a_{im} x^m).$$

$$= a_{li}\frac{\partial x^l}{\partial x_j} + a_{im}\frac{\partial x^m}{\partial x_j}$$

$$= a_{li}\delta^l_j + a_{im}\delta^m_j = a_{ji} + a_{ij} = 0$$

Q3/ (A) S is real and $S^T = -S$ (anti-) $A = (I-S)(I+S)^{-1}$

$$A^T A = [(I-S)(I+S)^{-1}]^T[(I-S)(I+S)^{-1}]$$

$$= [(I+S)^{-1}]^T (I-S)^T (I-S)(I+S)^{-1}$$

but $(I-S)^T = I^T - S^T = I^T + S$, then

$$A^T A = [(I+S)^{-1}]^T (I+S)(I-S)(I+S)^{-1}$$

$$= [(I+S)^T]^{-1}(I+S)(I-S)(I+S)^{-1}$$

$$= (I-S)^{-1}(I^2 - IS + IS - S^2)(I+S)^{-1} = (I-S)^{-1}(I^2 - S^2)(I+S)^{-1}$$

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$$= \underbrace{(I - S)^{-1}(I - S)}\,\underbrace{(I+S)(I+S)^{-1}}$$

$$= I\,I \Rightarrow A^T A = I \text{ (orthogonal)}.$$

Q3

(B) $\mathcal{L}\, 5e^{-3t}\sinh 2t$? , (b) $\mathcal{L}\, 2e^{3t}(4\cos 2t - 5\sin 2t)$?

$$\mathcal{L}\, 5e^{-3t}\sinh 2t = 5\mathcal{L}\{e^{-3t}\sinh 2t\} = 5\left(\frac{2}{(s-(-3))^2 - 2^2}\right)$$

$$= \frac{10}{(s+3)^2 - 2^2} = \frac{10}{s^2+6s+9-4} = \frac{10}{s^2+6s+5}$$

(b) $\mathcal{L}\{2e^{3t}(4\cos 2t - 5\sin 2t\} = 8\mathcal{L}\{e^{3t}\cos 2t\} - 10\mathcal{L}\{e^{3t}\sin 2t\}$

$$= \frac{8(s-3)}{(s-3)^2+2^2} - \frac{10(2)}{(s-3)^2+2^2} = \frac{8(s-3)-10(2)}{(s-3)^2+2^2}$$

$$= \frac{8s-44}{s^2-6s+13}$$

Q4 (A)

$$U^{\dagger} = U^{-1}, \quad A = A^{\dagger}, \quad B = B^{\dagger}, \quad AB = BA$$

$\Rightarrow (A+iB)^{\dagger}(A+iB) = I \Rightarrow (A^{\dagger} - iB^{\dagger})(A+iB) = I$

$$= A^{\dagger}A + iA^{\dagger}B - iB^{\dagger}A + B^{\dagger}B = I$$

$$= A^2 + iAB - iBA + B^2 = I$$

$$A^2 + \cancel{iAB} - \cancel{iAB} + B^2 = I \Rightarrow A^2 + B^2 = I$$

(B)

$$A_x' = A_x\cos\varphi + A_y\sin\varphi$$

$$A_y' = -A_x\sin\varphi + A_y\cos\varphi$$

$$\left.\begin{aligned} x_1' &= x_1\cos\varphi + x_2\sin\varphi \\ x_2' &= -x_1\sin\varphi + x_2\cos\varphi \end{aligned}\right\}$$