University of Kerbala
College of Science
Department of Physics

Stage: 4^{th}
Course Title: Math. Methods for Physics
Examiner: Mohammed Al-Kaabi
Time: 3 hours

Final Examination 2012-2013
Note: Number of questions (4) Answer (4) only

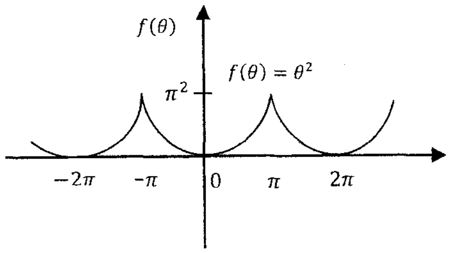
Q1. (A) The tension T_1, T_2 and T_3 in a simple framework are given by the equations: (6 Mark)

$$\begin{aligned} 5\,T_1 + 5\,T_2 + 5\,T_3 &= 7.0 \\ T_1 + 2\,T_2 + 4\,T_3 &= 2.4 \\ 4\,T_1 + 2\,T_2 &= 4.0 \end{aligned}$$

Determine T_1, T_2 and T_3 using determinant.

B. Determine the Laplace transforms of: (a) $\sin^2 t$ (b) $\cosh^2 3x$. (8 Mark)

Q2. Determine the Fourier series for the function $f(\theta) = \theta^2$ (shown in Fig. below) in the range $-\pi < \theta < \pi$. The function has period of 2π. (put $b_n = 0$) (12 Mark)



Q3. (A) Determine $\mathcal{L}^{-1}\left\{\frac{5s^2+8\,s-1}{(s+3)(s^2+1)}\right\}$. (6 Mark)

B. Prove that, if $\tilde{A}$ is Hermitian and $\tilde{U}$ is unitary then $\tilde{U}^{-1}\tilde{A}\,\tilde{U}$ is Hermitian. (6 Mark)

Q4. (A) Prove that, if $\tilde{A}$ is an orthogonal matrix, then $|\tilde{A}| = \pm 1$. (4 Mark)

B. If $\tilde{A} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ -1 & 2 & 0 & 1 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$ and the characteristic equation of $\tilde{A}$ is given by: (4 Mark)

$|\tilde{A} - \lambda\tilde{I}|=0$. Find the values of λ.

C. Expand the following: **a.** $a_{ij}x_j$ **b.** $\frac{\partial}{\partial x^i}\left(\sqrt{g}\,a^i\right)$ (4 Mark)

Good Luck

M. Al Kaabi
Signature of the Course Lecturer

N. ALDAHAN
Head of the Department signature

جامعة كربلاء كلية العلوم
رئيس القسم
قسم الفيزياء

اجوبة النموذج الأول - فيزياء رياضية / ٤ / قسم الفيزياء

للعام الدراسي 2012 - 2013

Q1/(A)

$$D = \begin{vmatrix} 5 & 5 & 5 \\ 1 & 2 & 4 \\ 4 & 2 & 0 \end{vmatrix} = 5(-8) - 5(-16) + 5(2-8) = -40 + 80 - 30 = 10$$

$$D_1 = \begin{vmatrix} 7 & 5 & 5 \\ 2.4 & 2 & 4 \\ 4 & 2 & 0 \end{vmatrix} = 7(-8) - 5(-16) + 5(4.8 - 8) = -56 + 80 - 16 = 8$$

$$T_1 = \frac{8}{10} = 0.8$$

$$D_2 = \begin{vmatrix} 5 & 7 & 5 \\ 1 & 2.4 & 4 \\ 4 & 4 & 0 \end{vmatrix} = 5(-16) - 7(-16) + 5(4 - 9.6) = -80 + 112 - 28 = 4$$

$$T_2 = \frac{4}{10} = 0.4$$

$$D_3 = \begin{vmatrix} 5 & 5 & 7 \\ 1 & 2 & 2.4 \\ 4 & 2 & 4 \end{vmatrix} = 5(8 - 4.8) - 5(4 - 9.6) + 7(2-3) = 16 + 28 - 42 = 2$$

$$T_3 = \frac{2}{10} = 0.2$$

(B) (a) $\sin^2 t$ (b) $\cosh^2 3x$ (1)

(a) since $\cos 2t = 1 - 2\sin^2 t$ thus $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$, Hence,

$$\mathcal{L}\{\sin^2 t\} = \mathcal{L}\left\{\tfrac{1}{2}(1 - \cos 2t)\right\} = \tfrac{1}{2}\mathcal{L}(1) - \tfrac{1}{2}\mathcal{L}\{\cos 2t\}$$

$$= \tfrac{1}{2}\left(\tfrac{1}{s}\right) - \tfrac{1}{2}\left(\frac{s}{s^2+2^2}\right)$$

$$= \frac{(s^2+4) - s^2}{2s(s^2+4)} = \frac{4}{2s(s^2+4)} = \frac{2}{s(s^2+4)}$$

(b) $\cosh^2 3x =$

$\cosh^2 x = \frac{1}{2}(1 + \cosh 2x)$ Hence, $\cosh^2 3x = \frac{1}{2}(1 + \cosh 6x)$, Thus

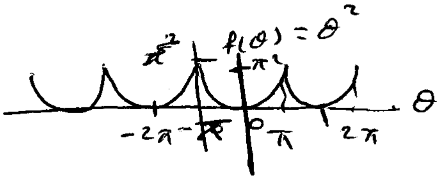
$$\mathcal{L}\{\cosh^2 3x\} = \mathcal{L}\left\{\tfrac{1}{2}(1 + \cosh 6x)\right\} = \tfrac{1}{2}\mathcal{L}(1) + \tfrac{1}{2}\mathcal{L}\{\cosh 6x\}$$

$$= \tfrac{1}{2}\left(\tfrac{1}{s}\right) + \tfrac{1}{2}\left(\frac{s}{s^2 - 6^2}\right)$$

$$= \frac{2s^2 - 36}{2s(s^2 - 36)} = \frac{s^2 - 18}{s(s^2 - 36)}$$

(4) عزيز

Q2/ $f(\theta) = \theta^2$



$f(\theta) = a_0 + \sum_{n=1}^{\infty} (a_n + b_n)$

$a_0 = \frac{1}{\pi}\int_0^{\pi} f(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} \theta^2 d\theta = \frac{1}{\pi}\left[\frac{\theta^3}{3}\right]_0^{\pi} = \frac{\pi^2}{3}$

$a_n = \frac{2}{\pi}\int_0^{\pi} f(\theta)\cos n\theta\, d\theta = \frac{2}{\pi}\int_0^{\pi} \theta^2 \cos n\theta\, d\theta = \frac{2}{\pi}\left[\frac{\theta^2 \sin n\theta}{n} + \frac{2\theta\cos n\theta}{n^2} - \frac{2\sin n\theta}{n^3}\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\left(0 + \frac{2\pi\cos n\pi}{n^2} - 0\right) - (0)\right] = \frac{4}{n^2}\cos n\pi$

when n is odd, $a_n = \frac{-4}{n^2}$, Hence, $a_1 = \frac{-4}{1^2}$, $a_3 = \frac{-4}{3^2}$, $a_5 = \frac{-4}{5^2}$ and so on

when n is even, $a_n = \frac{4}{n^2}$, Hence $a_2 = \frac{4}{2^2}$, $a_4 = \frac{4}{4^2}$, and so on.

Hence the forier series is,

$f(\theta) = \theta^2 = \frac{\pi^2}{3} - 4\left(\cos\theta - \frac{1}{2^2}\cos 2\theta + \frac{1}{3^2}\cos 3\theta - \frac{1}{4^2}\cos 4\theta + \frac{1}{5^2}\cos 5\theta - \dots\right)$

Q3/(A) Determine $\mathcal{L}^{-1}\left\{\frac{5s^2+8s-1}{(s+3)(s^2+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{2}{s+3} + \frac{3s-1}{s^2+1}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{2}{s+3}\right\} + \mathcal{L}^{-1}\left\{\frac{3s}{s^2+1}\right\} - \mathcal{L}\left\{\frac{1}{s^2+1}\right\}$

$= 2e^{-3t} + 3\cos t - \sin t$

Q3 B if $\hat{A}$ is hermitian and $\hat{U}$ is unitary the $U^{-1}AU$ is he

$U^{\dagger} = U^{-1}$, $A^{\dagger} = A$

$(U^{-1}AU)^{\dagger} = U^{\dagger}A^{\dagger}(U^{-1})^{\dagger} = U^{-1}A(U^{-1})^{-1} = U^{-1}AU$

Q4/(A) $A^{-1} = A^{T}$ (orthogonal), $A^{-1}A = I \Rightarrow |A^{T}A| = |I| \Rightarrow |A^{T}||A| = 1$

$\Rightarrow$ also $|A^{T}| = |A| \Rightarrow |A||A| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \mp 1$

Q4/(B) $\hat{A} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ -1 & 2 & 0 & 1 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix} \Rightarrow \left|\begin{pmatrix} 0 & 0 & 1 & 1 \\ -1 & 2 & 0 & 1 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix}\right| = 0$

$\begin{vmatrix} -\lambda & 0 & 1 & 1 \\ -1 & 2-\lambda & 0 & 1 \\ -1 & 0 & 2-\lambda & 1 \\ 1 & 0 & -1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda_1 = 0,\ \lambda_2 = 1,\ \lambda_3 = -1,\ \lambda_4 = 2$

Q4/ (c) (a) $a_{ij} x^j$ Expand $\Rightarrow$ $a_{i1} x_1 + a_{i2} x_2 + \cdots a_{in} x^n$

(b) $\frac{\partial}{\partial x^i}(\sqrt{g}\, a^i) = \frac{\partial}{\partial x^1}(\sqrt{g}\, a^1) + \frac{\partial}{\partial x^2}(\sqrt{g}\, a^2) + \ldots \frac{\partial}{\partial x^n}(\sqrt{g}\, a^n).$