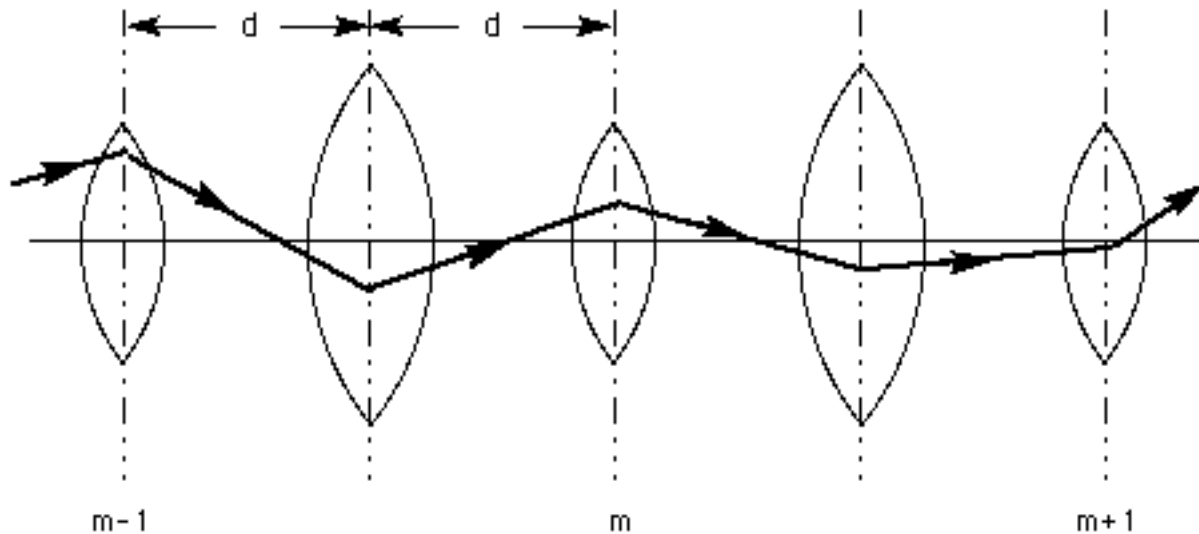


IV. OPTICAL RESONATORS:

STABILITY CRITERIA FOR RESONATORS AND PERIODIC OPTICAL STRUCTURES BY RAY OPTIC ANALYSIS

Consider a prototypical periodic guiding lens system or an equivalent resonator.



Using the appropriate **ABCD** matrix with the indicated reference planes ¹⁷, we may write

$$r_{m+1} = A r_m + B \theta_m \quad [IV-1a]$$

and

$$r_{m+1} = C r_m + D \theta_m \quad [IV-1b]$$

¹⁷ Where for reference, we see that

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d & 1 & 0 & 1 & d & 1 & 0 \\ 0 & 1 & -1/f_2 & 1 & 0 & 1 & -1/f_1 & 1 \end{pmatrix} = \begin{pmatrix} 1-d/f_2-2d/f_1+d^2/f_1f_2 & 2d-d^2/f_2 \\ d/f_1f_2-1/f_1-1/f_2 & 1-d/f_2 \end{pmatrix}$$

From the first equation we write

$$a_m = \frac{a_{m+1} - A}{B} \quad \text{and} \quad a_{m+1} = \frac{a_{m+2} - A}{B}.$$

and substitute into Equation [IV-1b] to obtain

$$a_{m+2} - [A + D] a_{m+1} + [AD - BC] a_m = 0 \quad [\text{IV-2a}]$$

The determinant of the coefficients $|A \ D - BC| = 1$ so that

$$a_{m+2} - [A + D] a_{m+1} + a_m = 0. \quad [\text{IV-2a}]$$

We see that

$$\frac{A+D}{2} = 1 - \frac{d}{f_2} - \frac{d}{f_1} + \frac{d^2}{2f_1f_2} = -1 + 2 \left(1 - \frac{d}{2f_1}\right) \left(1 - \frac{d}{2f_2}\right) \quad [\text{IV-3}]$$

Thus, stable ray propagation may be characterized by **bound solutions** of the form

$a_m = a_0 \exp(i m \theta)$ which are possible if and only if

$$\exp(i\theta) + \exp(-i\theta) = 2 \cos \theta = A + D = 2 \quad \text{or} \quad \cos \theta = -1 \quad [\text{IV-4}]$$

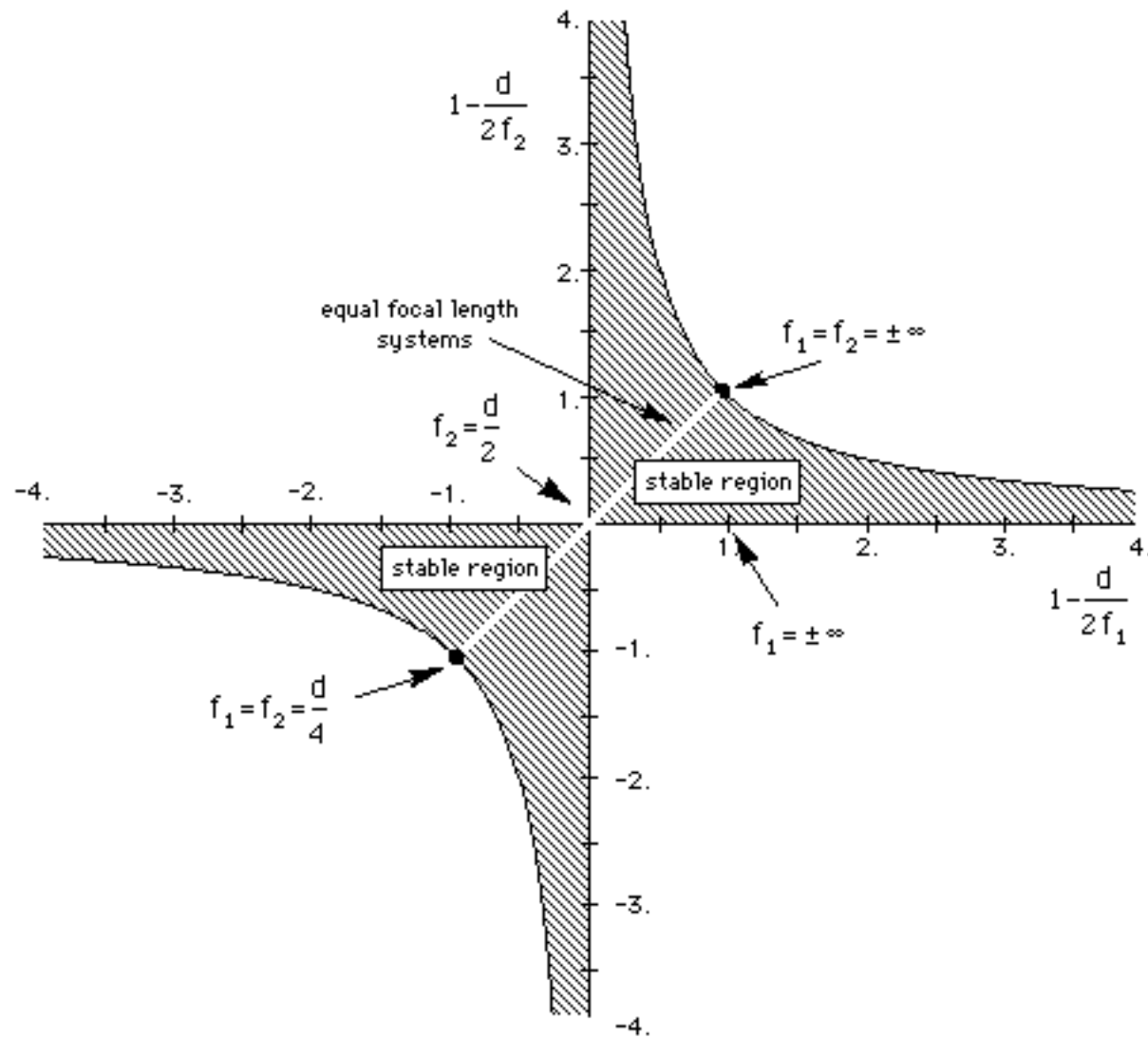
Therefore propagation is stable -- *i.e.* the rays are confined -- when $|\cos \theta| \leq 1$ so that

$$0 \leq 1 - \frac{d}{2f_1} - \frac{d}{2f_2} + \frac{d^2}{2f_1f_2} \leq 1. \quad [\text{IV-5}]$$

Ray stability of rays in a periodic system may be usefully characterized in terms of the

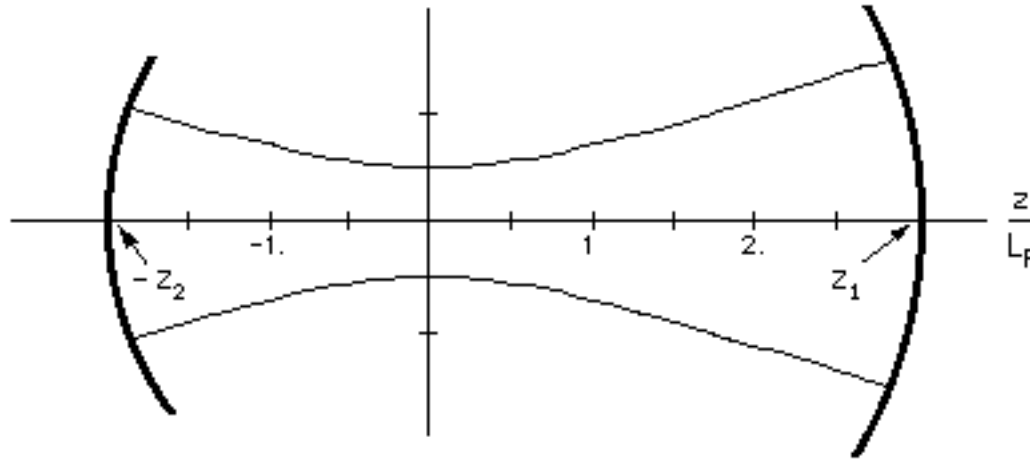
variables $u_1 = 1 - \frac{d}{2f_1}$ and $u_2 = 1 - \frac{d}{2f_2}$ as follows:

Stability (Confinement) Diagram for Periodic Systems



STABILITY OF A SPHERICAL MIRROR RESONATORS -- USING SOLUTIONS OF THE PARAXIAL WAVE EQUATION:

Consider a Hermite-Gaussian mode confined in an **asymmetrical spherical cavity**:



In order to sustain a resonant mode in such a cavity, the beam's radius of curvature must match each mirror's radius of curvature at the mirror's surface and, thus, the following conditions must hold (see Equation [III-20]):

$$R_1 = z_1 + \frac{L_F^2}{z_1} \quad \text{and} \quad R_2 = -z_2 - \frac{L_F^2}{z_2} \quad [\text{IV-5a}]$$

where $d = z_1 + z_2$. Hence, we see that

$$z_1 = \frac{R_1 \pm \sqrt{R_1^2 - 4L_F^2}}{2} \quad \text{and} \quad z_2 = \frac{-R_2 \pm \sqrt{R_2^2 - 4L_F^2}}{2} . \quad [\text{IV-5b}]$$

with a lot of algebra we can show that

$$L_F^2 = \frac{w^2(0)^2}{[u_1 R_1 - u_2 R_2]^2} = \frac{-du_1 u_2 R_1 R_2 [d + u_1 R_1 - u_2 R_2]}{[u_1 R_1 - u_2 R_2]^2} \quad [\text{IV-6}]$$

where now $u_1 = 1 - d/R_1$ and $u_2 = 1 + d/R_2$.

For a **symmetric resonator** $R_2 = -R_1$ and $u_1 = u_2$

$$L_F^2 = \frac{w^2(0)^2}{4} = \frac{d[2R - d]}{4} \quad [\text{IV-7a}]$$

and

$$w(z_1) = w(-z_2) = \frac{d}{2} \frac{2R^2}{d(R - d/2)} \quad [\text{IV-7b}]$$

For an **asymmetric resonator**, it can be shown with a bit more algebra that

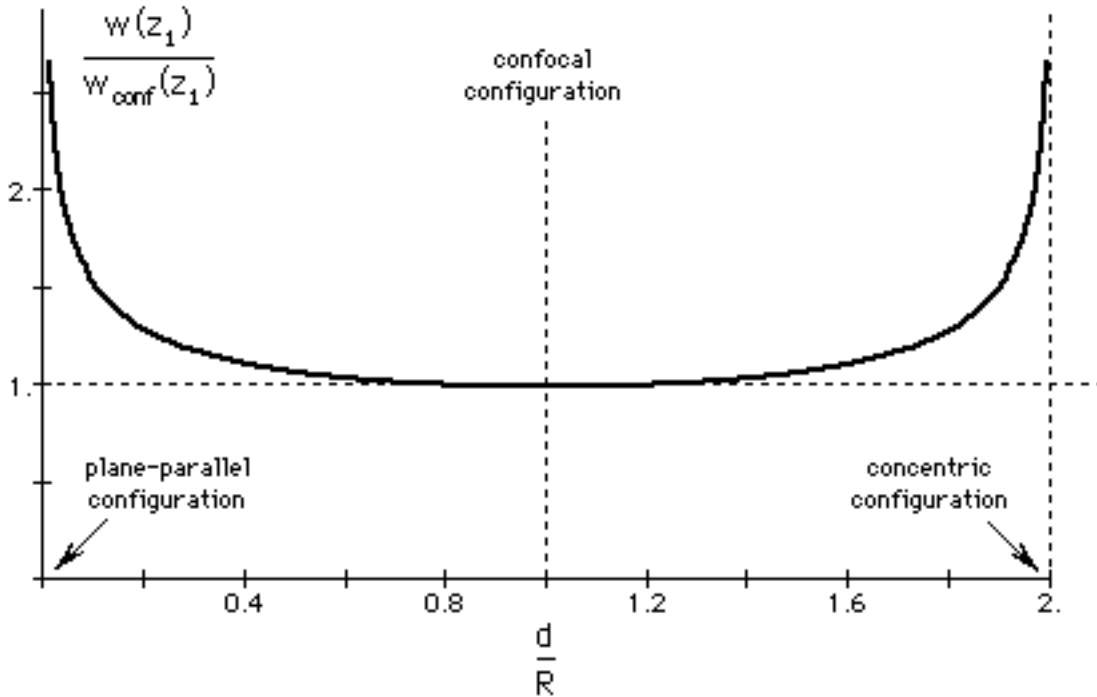
$$w(z_1) = \frac{d}{u_1(1 - u_1 u_2)} \quad [\text{IV-8a}]$$

$$w(-z_2) = \frac{d}{u_2(1 - u_1 u_2)} \quad [\text{IV-8b}]$$

As a measure of the effect of resonator length and mirror radius on **diffraction loss** consider the ratio:

$$\frac{w(z_1)}{w_{\text{conf}}(z_1)} = \frac{w(-z_2)}{w_{\text{conf}}(-z_2)} = \frac{d}{R} \left(1 - \frac{d}{2R}\right)^{-1/4} \quad [\text{IV-9}]$$

where $w_{\text{conf}}(z_1) = \sqrt{2} w(0)$ is beam width at the mirror for the confocal configuration --
i.e., when both mirrors have their focal points at the mid-point of the cavity



Resonance Frequencies of the Optical Resonator:

From Equation [III-28], we see the the “round-trip” cold resonance condition for a Hermite-Gaussian mode is given by

$$k d - (n + m + 1) \left[\tan^{-1} \left(z_1 / L_F \right) + \tan^{-1} \left(z_2 / L_F \right) \right] = N \quad [\text{IV-10a}]$$

where N is an integer. In terms of frequency, the resonance condition is

$$= \frac{c}{d} \left\{ N + (n + m + 1) \left[\tan^{-1}(z_1 / L_F) + \tan^{-1}(z_2 / L_F) \right] \right\} \quad [\text{IV-10b}]$$

After much algebra, it can be shown that:

$$= \frac{c}{d} \left[N + (n + m + 1) \cos \sqrt{u_1 u_2} \right] \quad [\text{IV-10b}]$$