**3) Baysion Estimation (fixed point) : طريقة بيز**

ناقشنا طريقتين لتقدير المعلمة المجهولة هما طريقة العزوم ودالة الامكان الاعظم وفي هاتين الطريقتين تم افتراض ان المعلمة المجهولة هي نقطة ثابتة اما في طريقة تقدير بيز فبالإضافة الى العينة العشوائية فيتم افتراض ان المعلمة المجهولة متغير عشوائي هذا يعني وجود معلومات اضافية حول المعلمة .

والآن يمكن التعامل مع المتغيرات العشوائية كما في التعاريف التالية :

**Def. 5)** represents the joint p.d.f. of two random variables and .

**Def. 6)** represents the conditional p.d.f. of given is held fixed .

**Def. 7)** represents the marginal p.d.f. of parameter which is called prior dist. of .

**Def. 8)** represents the conditional density of given which is called the posterior dist. of given .

**Remark 3)**

**Def. 9)** Consider estimating . let denote an estimate of . the loss func. measure the penalty that a rises from taking action when is true value of the parameter .

**Def. 10)** Risk func. Is given by the formula where the expectation is taken with respect to the distribution of the random variables with fixed .

**Bayes Method**

Let be a r.s. from the p.d.f. of and the prior dist. of , and the loss function is , the Bayes estimate is the value of which minimizes the equation

that is ; the Bayes estimator of is , is solution of the equation .

**Steps of Solution**

1. and are given
2. Find
3. Find
4. Solve

**Ex (9) :** Let be a r.s. from , If the prior dist. of is , let find the Bayes estimator of .

**مثال :** ليكن متغير عشوائي بتوزيع بواسون بمعلمة والتقدير الاولي للمعلمة المجهولة ودالة الخسارة أوجد الى متغيرات عشوائية مستقلة متماثلة من توزيع بواسون بمعلمة .

**Q :** let be a r.s. from . let . If find the Bayes estimator .

**3-4 Properties of Good Estimator : مواصفات المقدر الجيد**

**1) Un biasedness :عدم التحيز**

Let be a random sample from a population having p.d.f. , with unknown parameter . let be an estimator of . therefore , is said to be unbiased estimator of , iff other wise , the bias of an estimator can be defined as

Generally we can say that is an unbiased estimator of iff

**Ex (10) :** Let be a random sample from shaw that

1. is unbiased of
2. is biased of .

**2) Mean square error : متوسط مربعات الخطأ**

If is an estimator of the unknown parameter , then the mean square error can be defined as

To be a good estimator of we need to have the msE is small as possible therefore , can be small .

**Ex (11) :** A r.s. of size(n) from the p.d.f. compare between the two estimators and of .

**3) Sufficiency : الكفاية**

**Theorem :** Let denote a random sample from dist. has p.d.f. , let be statistic whose p.d.f. is , then is a sufficient statistic for if and only if

Where for every fixed value of

Does not depend upon

**Ex (12) :** A r.s. of size(n) from the p.d.f. is the m.l.e. of (prove that) show is suff .

**Ex (13) :** be a r.s. from show that is sufficient stat. for .

**Q :** solve 7.6 , 7.7 , 7.10 , 7.11 , 7.12 p.222

**Facterization Theorem :** Let be a r.s. of size(n) with p.d.f. where may be a vector . the statistics are Jointly sufficient iff the joint density of can be factored as :

Where the non-negative func. depends on only through the stat. and parameter . And the non-negative func. is independent of .

**Ex (14) :** be a r.s. from the dist. with p.d.f. find suff. Stat. of .

**Ex (15) :** A r.s. of size(n) from find Jointly sufficient statistics for and ?