GENERAL TOPOLOGY

2013-2014

((***Topological spaces***)) : الفضاءات التبولوجية

Let X be a non-empty set. A class τ of subsets of X is a topology on X if and only if τ satisfies the following axioms.

i. X and $\emptyset$ belong to τ .
ii. The intersection of any two sets in τ belong to τ .
iii. The union of any number of sets in τ belong to τ.

The members of τ are the called open sets, and the pair (X,τ) is called a topological space.

Example 1: Consider the following classes of subsets of $X=\{1,2,3\}$

$\tau_1=\{X,\emptyset,\{1\},\{2\},\{1,2\}\}$ $\tau_2=\{X,\{1\},\{2\},\{1,2\}\}$

$\tau_3=\{X,\emptyset,\{1,2\},\{1,3\}\}$ $\tau_4=\{X,\emptyset,\{1\},\{2\}\}$

$\tau_5=\{X,\emptyset,\{1,3\}\}$ $\tau_6=\{X,\emptyset\}$

Then τ_1, τ_5 and τ_6 are topologies on X .

But τ_2 is not a topology on X since the $\emptyset\notin\tau_2$.

Also, τ_3 is not a topology on X since the intersection $\{1,2\}\cap\{1,3\}=\{1\}\notin\tau_3$.

And τ_4 is not a topology on X since the union $\{1\}\cup\{2\}=\{1,2\}\notin\tau_4$.

Remarks:

1. There are 29 topology on a set of three element. Check?
2. $\tau_I=\{X,\emptyset\}$ is called ***Indiscrete topology*** on X. (التبولوجيا الاخشن)
3. $\tau_D=\mathrm{P}(X)$ [power of X] is called ***Discrete topology*** on X.(التبولوجيا الانعم)
4. There are more than one topology on a set X, with more than one element.
5. The intersection $\tau_1\cap\tau_2$ of any two topologies $\tau_1 and\ \tau_2$ on X is also a topology on X. check?

Example 2: Consider the following classes of subsets of $X = \{a, b, c, d, e\}$

$$\tau_1 = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$$

$$\tau_2 = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\tau_3 = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{a, b, d, e\}\}$$

Then τ_1 is a topology on X since it satisfies the necessary

three axioms [i],[ii] and [iii].

But τ_2 is not a topology on X since the union

$\{a, c, d\} \cup \{b, c, d\} = \{a, b, c, d\} \notin \tau_2$, i.e. τ_2 dose not satisfy the axiom [iii].

Also, τ_3 is not a topology on X since the intersection

$\{a, c, d\} \cap \{a, b, d, e\} = \{a, d\} \notin \tau_3$, i.e. τ_3 dose not satisfy the axiom [ii].

Example 3: Let τ_u denote the class of all open sets of real numbers. Then τ_u is a topology on ***R***; it is called ***usual topology*** on ***R***. (التبولوجيا الحقيقية)

i.e. $\tau_u = \{u \subseteq R : u\ is\ an\ open\ interval\ or\ the\ union\ of\ open\ interval\}$

Question1: Is $\tau = \{A_n / A_n = \{1,2, \ldots, n\}\} \cup \{\mathbb{N}, \emptyset\}$ topology on $\mathbb{N}$.

Sol:

$A_1 = \{1\}$, $A_2 = \{1,2\}$, $A_3 = \{1,2,3\}$ ....

i. $\emptyset$ and $\mathbb{N}$ belong to τ.

ii. Let A_i and A_j belong to τ, to show $A_i \cap A_j$ belong to τ .
If $i = j\ or\ i < j\ then\ A_i \cap A_j = A_i \in \tau$.
Or if $i > j\ then\ A_i \cap A_j = A_j \in \tau$

iii. Let $\{A_n\}_{n \in \mathbb{N}}$ be a family of elements of τ, to show $\bigcup_n A_n \in \tau$
But $\bigcup_n A_n = \mathbb{N} \in \tau$

So τ is a topology on $\mathbb{N}$.

Question2: Is $\tau = \{A_n / A_n = \{n, n+1, n+2, \ldots\}\} \cup \{\emptyset\}$ topology on $\mathbb{N}$. check?